Trade and Real Wages of the Rich and Poor: Cross-Country Evidence *

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Abstract

Trade liberalization affects real-wage inequality through two channels: the distribution of nominal wages across workers and, if the rich and the poor consume different bundles of goods, the distribution of price indices across consumers. I provide a unified framework incorporating both channels by allowing for non-homothetic preferences and worker heterogeneity across jobs. Because skill-intensive goods are also high-income elastic in the data, I find an intuitive, previously unexplored, and strong interaction between the two channels. I parametrize the model for 40 countries using sector-level trade and production data, and find that trade cost reductions decrease the relative nominal wage of the poor and the relative price index for the poor in all countries. On net, real-wage inequality falls everywhere.

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1 Introduction

Trade liberalization may impact an individual's real wage through her nominal wage and her consumer price index. The change in her nominal wage depends on changes in producer prices and the job in which she is employed, where the job of her employment is determined by her characteristics such as age, gender and educational attainment. On the other hand, the change in her consumer price index depends on changes in prices of the basket of goods that she consumes, where her consumption basket is determined by her nominal wage in addition to prices. A vast majority of the literature focuses on the effect of trade on the distribution of nominal wages. A small number of studies consider its differential impact on consumer price indices. In this paper, I provide a unified framework that incorporates both the *expenditure channel*, i.e., changing consumer price indices, and the *income channel*, i.e., changing nominal wages, to measure the distributional effects of trade in a large cross-section of countries.[1]

I build a model combining demand heterogeneity across consumers with productivity heterogeneity across workers. On the demand side, I use the Almost Ideal Demand System (AIDS) to capture non-homothetic preferences. This demand specification allows the consumption baskets of high-income and low-income individuals to differ so that price changes resulting from trade liberalization have a differential impact on their consumer price indices. On the supply side, I use an assignment model of the labor market parametrized with a Fréchet distribution to capture heterogeneity of workers across jobs. Individuals have comparative advantage across sectors—based on their age, gender and educational attainment—and, therefore, sort into different sectors. Consequently, price changes resulting from trade liberalization have a differential impact on individuals' nominal wages depending on the sectors in which they work. In addition, I also allow individuals to differ in their absolute advantage such that labor groups differ in their average productivity and, therefore, have different nominal wages regardless of individuals' sectoral choices.[2] This assumption generates a potential link between the skill distribution and the wage distribution and, as a result, a potential correlation between the change in an individual's nominal wage and the change in her consumer price index.

In isolation, these two channels have well-understood implications. Shutting down the expenditure channel, I find that the income channel benefits the poor more than the rich in low-income countries and the rich more than the poor in high-income countries. This

[1] I focus on labor earnings, which are the main source of income for most people.

[2] Workers in a labor group share the same observable characteristics such as age, gender and educational attainment.

is consistent with standard factor proportions theory in which a reduction in trade costs raises the relative nominal wage of the abundant factor in every country, benefiting the unskilled (and poor) workers in skill-scarce countries that are low income and the skilled (and rich) workers in skill-abundant countries that are high income. Shutting down the income channel, I find that the expenditure channel benefits the poor more than the rich in every country and more so in high-income countries. Intuitively, lower trade costs increase real incomes and, therefore, decrease the relative demand for and the relative price of low-income elastic goods. Because low-income consumers spend more on these goods, they benefit relatively more. The expenditure channel benefits the poor relatively more in high-income countries because these countries are net importers of low-income elastic goods.

These two channels do not work in isolation. Studying either channel in the absence of the other leads to profoundly biased results qualitatively and quantitatively. Specifically, their interaction implies that the income channel benefits the rich in every country, which is consistent with a large body of empirical evidence; see e.g. Goldberg and Pavcnik (2007). Intuitively, when both channels are active, lower trade costs increase real incomes and, therefore, decrease the relative demand for and the relative price of low-income elastic goods as discussed above.[3] Since the poor disproportionately produce unskill-intensive goods, which are low-income elastic, their relative nominal wage falls in every country. This effect is absent when only the income channel is active. Moreover, the interaction of these two mechanisms also implies that the poor's relative benefit from the expenditure channel is magnified in every country. Intuitively, because nominal wage inequality rises in every country, as just described, the relative demand for and the relative price of low-income elastic goods fall even further, reducing the relative price index for the poor in every country. This effect is absent when only the expenditure channel is active because nominal wage inequality is constant in that case.[4]

[3]Alternative models can also generate the increase in the skill premium, for example, a close-economy macroeconomic model where there is a uniform increase in productivity with non-homothetic preferences. Caron et al. (2014) find that it raises wages of skilled workers significantly, increasing the nominal wage inequality at equilibrium. In this paper, I focus on the impact of a decrease in trade costs as an explanation.

[4]There are other types of gains from trade. For example, the access to more product varieties makes everyone better off. Fajgelbaum and Khandelwal (2016) find that low-income consumers spend relatively more on sectors that are more traded, where high-income consumers spend relatively more on services, which are among the least internationally traded sectors. As a result of trade liberalization, there is a bigger increase in product varieties in the sectors that have a low income elasticity, which benefits the poor even more. On the other hand, Faber and Fally (2016) find that changes in product varieties affect the price indices of rich and poor households asymmetrically. More product entry benefits richer households slightly more due to higher estimated love of variety. Consequently, it becomes an empirical

I parametrize the model for a sample of 40 countries (27 European countries and 13 other large countries) and 35 sectors using a range of datasets including the World Input-Output Database (WIOD) and the Integrated Public Use Microdata Series, International (IPUMS-I). WIOD provides information on bilateral trade flows and production data.[5] I derive a sectoral non-homothetic gravity equation that allows me to estimate the elasticity of substitution and the income elasticity of goods as follows.[6] First, I estimate the elasticity of substitution by projecting countries' sectoral expenditure shares on trade costs. Second, I estimate the income elasticity of each good using the following insight: if high-income or more unequal countries spend relatively more on a good, then I infer that this good is high-income elastic. IPUMS-I provides publicly available nationally representative survey data for 82 countries that are coded and documented consistently across countries and over time. It reports individual-level information including age, gender, educational attainment, labor income and sector of work. This rich database enables me to estimate the Fréchet dispersion parameter of the within-group distribution of efficiency units across sectors which determines the extent of worker reallocation and, thus, the responsiveness of group average wages to changes in sectoral output prices. In addition, I am able to estimate the comparative advantage of different labor groups across sectors based on observed worker sorting patterns. Intuitively, if a worker type (relative to another worker type) is more likely to sort into a sector (relative to another sector), then I infer that they are relatively more productive in that sector. Using the estimates of group average wages and other parameters, I can back out the absolute advantage of different labor groups.

With these parameter estimates, I conduct two counterfactual analyses to quantify the distributional effects of trade liberalization. To demonstrate how the model works, I begin with a simple counterfactual exercise in which I consider a 5% reduction in all bilateral trade costs. I find that the average welfare gain across the 40 countries is about 1.2%, which is in line with the previous literature that abstracts from relative effects across individuals within countries.[7] Within each country, as one moves up the initial nominal wage distribution, gains decline. Specifically, moving up from one decile to the next

question which factor dominates.

[5]One important feature of the WIOD is that it includes the input-output transactions of a country with itself. Typically, the domestic market accounts for the large marjority of demand for most production.

[6]The sectoral non-homothetic gravity equation based on the AIDS was first dervied in Fajgelbaum and Khandelwal (2016). However, their model assumptions imply that the change in income is 0 for all consumers.

[7]Eaton and Kortum (2002) consider a counterfactual where the 19 OECD countries collectively remove the 5 percent tariff on all imports and find that most countries gain around one percent.

reduces gains by 0.1 percentage point: the bottom 10th percentile experiences a real wage gain that is larger than the top 10th percentile in every country, and the difference is 0.8 percentage points in the average country. These results highlight that the distributional effects of trade liberalization are large compared to its average effect. I obtain the result that the poor gain relative to the rich in spite of the fact that I find the opposite result for nominal wages. In the average country, the bottom 10th percentile see their nominal wages decrease by 0.2 percentage points relative to the top 10th percentile. Hence, the reduction in the poor's relative price index must fall substantially. In the average country, the bottom 10th percentile see their consumer price indices decrease by 1 percentage point more than the top 10th percentile.

My framework also allows me to re-examine the impact of a significant increase in U.S. manufacturing imports from China on U.S. real-wage inequality while accounting for both channels and their interaction.[8][9] I consider a uniform reduction in trade costs between the U.S. and China that would yield a $1000 per U.S. worker increase in Chinese manufacturing imports. I find that this reduction in trade costs decreases the consumer price index for a U.S. representative consumer by 0.85%. An individual whose nominal wage is at the 10th percentile of the initial distribution sees a further 0.35 percentage point reduction in her consumer price index compared to the representative consumer, while an individual whose nominal wage is at the 90th percentile sees her consumer price index decrease by 0.1 percentage point less than the representative consumer. This result arises because Chinese manufacturing goods are low-income elastic and, consequently, their lower prices benefit more the poor individuals who spend relatively more on these goods. Although the former sees a bigger decline in her nominal wage (0.13% vs. 0.11%) because she's more likely to work in manufacturing sectors that are in direct competition with cheaper Chinese imports, this income effect is more than offset by her much lower consumer price index. Rising Chinese import competition increases the real wage of the poor by 0.43 percentage points more than that of the rich in the U.S.

A vast body of research has examined the impact of trade on the distribution of earnings across workers. Most recently, Galle, et al. (2015) develop the notion of "risk-adjusted gains from trade" to evaluate the full distribution of welfare changes in one

[8]Autor et al. (2013), Autor et al. (2014) and Acemoglu et al. (2016) study the impact of increased Chinese import competition on employment and earnings of U.S. workers by comparing more affected industries and local labor markets to less affected ones but have no implications at the aggregate level.

[9]Another interesting counterfactual to consider is the Trans-Pacific Partnership (TPP). I can use my framework to simulate the aggregate and distributional effects of this trade agreement for each of the participating countries.

measure which generalizes the specific-factors intuition to a setting with endogenous labor allocation. Similarly, I focus on changes in relative nominal wages across labor groups that result from changes in relative demand across sectors driven by international trade.[10] There is a small number of studies that have considered price indices as a channel through which trade liberalization can affect inequality. For example, Fajgelbaum and Khandelwal (2016) develop a methodology to measure the unequal gains from trade through the expenditure channel using only aggregate statistics. I extend this approach to incorporate the differential impact of trade liberalization on individuals' nominal wages. In contrast, Faber (2014) exploits barcode level microdata from the Mexican Consumer Price Index and studies the relative price effect of NAFTA on the differential change in the cost of living between rich and poor households. Faber and Fally (2016) use detailed matched U.S. home and store scanner microdata to explore the implications of firm heterogeneity for household price indices across the income distribution. I complement the existing literature by incorporating both the expenditure and the income channels as well as their interaction in a unified framework to analyze the heterogeneous impact of counterfactual trade shocks across individuals in a large set of countries.

To my knowledge, there are only three case studies that have looked at these two channels jointly.[11] Porto (2006) studies the distributional effects of Mercosur, a regional trade agreement among Argentina, Brazil, Paraguay and Uruguay, during the 1990s. Nicita (2009) extends Porto's approach by adding a link from trade policy to domestic prices and studies the trade liberalization that took place in Mexico during the period 1990-2000. Marchand (2012) allows the tariff pass-through to differ across geographical regions and studies the trade reforms in India between 1988 and 2000. The structure of my model allows me to estimate the effects for more countries. By looking at a wide

[10] See also Adão (2015), Burstein, Morales and Vogel (2016) and Dix-Carneiro and Kovak (2015). I don't incorporate some of the mechanisms that have been studied in the literature linking international trade to inequality through the earnings channel. For example, Yeaple (2005), Verhoogen (2008), Bustos (2011), Burstein and Vogel (2016) and Bloom, et al. (2015) show that trade liberalization increases the measured skill bias of technology by reallocating resources from less to more skill-intensive firms within industries and/or inducing firms to increase their skill intensity. A major difficulty is the lack of a comprehensive, matched employer-employee dataset in many countries that covers the period of rising inequality which is usually confidential. In addition, these papers highlight the role of firms because the standard neoclassical theory of trade is inconsistent with the empirical finding that nominal wage inequality goes up everywhere in response to trade liberalization. I show that in a neoclassical setting, non-homothetic preferences allow the model to be fully consistent with the data. Therefore, it becomes an open question which mechanism is more important.

[11] Atkin et al. (2015) draw on a new collection of Mexican microdata to estimate the effect of foreign supermarket entry on household welfare. They do consider both the price index effect and the income effect, but focus only on the gains from retail FDI.

range of countries, I am able to identify general patterns across countries with different characteristics. I am also able to conduct model-based counterfactuals of different trade shocks which are important for policymakers. In addition, as critiqued in Goldberg and Pavcnik (2007), the predictions of these studies depend in a crucial way on estimates of the degree of pass-through from trade policy changes to product prices as well as the wage-price elasticities. These are difficult to estimate consistently with time-series data on wages and prices in a setting when many other policies change contemporaneously with trade.

The remainder of the paper proceeds as follows. In Section 2, I describe my multi-sector Armington model of trade with non-homothetic preferences on the demand side and heterogeneous labor with comparative advantage across sectors on the supply side. I study these two channels and their interaction analytically in Section 3. Section 4 contains a description of the data, and estimation strategy and results are gathered in Section 5. In Section 6, I discuss my counterfactual results. Section 7 concludes.

2 The Model

2.1 The Environment

I study an economy with N countries indexed by $n \in \mathcal{N} = \{1, \ldots, N\}$ and J final good sectors indexed by $j \in \mathcal{J} = \{1, \ldots J\}$. Each good is defined as a sector-country of origin pair. Within any $(j, n) \in \mathcal{J} \times \mathcal{N}$, output is homogeneous, and the market is perfectly competitive. In country n, there is a continuum of heterogeneous workers indexed by $z \in \mathcal{Z}^n$ with measure L^n. They are grouped into a finite number of types indexed by $\lambda \in \Lambda$ with measure $L^n(\lambda)$ based on observable characteristics: age, gender and education. I assume that types are mutually exclusive: $\mathcal{Z}^n(\lambda) \cap \mathcal{Z}^n(\lambda') = \emptyset$, $\forall \lambda \neq \lambda'$.

2.2 Definition of Welfare Change

Consider home country h. Trade liberalization induces a set of infinitesimal changes in log prices, $\left\{\widehat{p^h_{(j,n)}}\right\}_{(j,n)\in\mathcal{J}\times\mathcal{N}}$, and log wages, $\{\widehat{w_z}\}_{z\in\mathcal{Z}^h}$.[12] I define the local welfare change of individual z as the equivalent variation associated with this set of changes:[13]

[12] $\widehat{p^h_{(j,n)}} \equiv \partial \ln(p^h_{(j,n)})$ is the infinitesimal change in the log prices and $\widehat{w_z} \equiv \partial \ln(w_z)$ is the infinitesimal change in the log wages.

[13] Please see Appendix 9.1 for the derivation of the local welfare change as the equivalent variation.

$$\widehat{u_z} = \sum_j \sum_n s^z_{(j,n)} \left(\widehat{-p^h_{(j,n)}}\right) + \widehat{w_z} \tag{1}$$

Here, $s^z_{(j,n)}$ is the initial individual expenditure share on good (j,n). An individual's welfare is affected in two ways. The first is the change in her cost of living resulting from the change in prices which I refer to as the expenditure effect. Specifically, it is the price change applied to the pre-shock expenditure shares. A decrease in prices reduces the cost of living, and therefore increases an individual's welfare. The second is the change in her nominal wage which I refer to as the income effect.

I can further decompose the local welfare change of individual z into three components:

$$\begin{aligned} \underbrace{\widehat{u_z}}_{\text{total effect}} &= \underbrace{\sum_j \sum_n s^z_{(j,n)} \left(\widehat{-p^h_{(j,n)}}\right)}_{\text{expenditure effect}} + \underbrace{\widehat{w_z}}_{\text{income effect}} \\ &= \underbrace{\sum_j \sum_n S^h_{(j,n)} \left(\widehat{-p^h_{(j,n)}}\right)}_{\text{agg. exp. effect}\equiv\widehat{E^h}} + \underbrace{\sum_j \sum_n \left(s^z_{(j,n)} - S^h_{(j,n)}\right) \left(\widehat{-p^h_{(j,n)}}\right)}_{\text{ind. exp. effect}\equiv\widehat{\psi_z}} \\ &\quad + \underbrace{\widehat{w_z}}_{\text{income effect}} \end{aligned} \tag{2}$$

$\widehat{u_z} = \widehat{E^h} + \widehat{\psi_z} + \widehat{w_z}$, that is, the total effect is the sum of the aggregate expenditure effect, $\widehat{E^h}$, the individual expenditure effect, $\widehat{\psi_z}$ and the income effect, $\widehat{w_z}$. $S^h_{(j,n)}$ is country $h's$ aggregate expenditure share on good (j,n). The aggregate expenditure effect can be thought of as the impact of trade liberalization on the cost of living under homothetic preferences where the ratios of goods demanded by consumers depend only on relative prices, not on income or scale. This effect is the same across all individuals within a country h. On the other hand, the individual expenditure effect implies that if individual z spends more on good (j,n), then the price decrease of that good increases her welfare by a larger amount.

2.3 Non-homothetic Preferences

I use the Almost-Ideal Demand System (AIDS) to capture the non-homotheticity in consumer preferences. It gives an arbitrary first-order approximation to any demand system and satisfies the axioms of order, aggregates over consumers without invoking parallel linear Engel curves, is consistent with budget constraints, and is simple to estimate.

The AIDS allows consumption baskets of high-income and low-income individuals to differ so that price changes resulting from trade liberalization have a differential impact on their consumer price indices. It belongs to the family of Log Price-Independent Generalized Prefereces defined by the following indirect utility function:

$$v\left(w_z, \mathbf{p}^h\right) = F\left[\left(\frac{w_z}{a\left(\mathbf{p}^h\right)}\right)^{1/b\left(\mathbf{p}^h\right)}\right] \tag{3}$$

where $F[\cdot]$ is a continuous, differentiable, and strictly increasing function. The AIDS is the special case that satisfies:

$$\begin{aligned} a\left(\mathbf{p}^h\right) &= \exp\left\{\underline{\alpha} + \sum_j \sum_n \alpha^h_{(j,n)} \ln p^h_{(j,n)} + \right. \\ &\left. \frac{1}{2}\sum_j \sum_n \sum_{j'} \sum_{n'} \gamma_{(j,n)(j',n')} \ln p^h_{(j,n)} \ln p^h_{(j',n')}\right\} \end{aligned} \tag{4}$$

$$b\left(\mathbf{p}^h\right) = \exp\left\{\sum_j \sum_n \beta_{(j,n)} \ln p^h_{(j,n)}\right\} \tag{5}$$

where $a\left(\mathbf{p}^h\right)$ is a homothetic price aggregator which captures the cost of a subsistence basket of consumption goods. $\underline{\alpha}$ is the outlay required for a minimal standard of living when prices are unity. $\alpha^h_{(j,n)}$ is importer h's taste for good (j,n). $\gamma_{(j,n)(j',n')}$ is the cross elasticity between two goods (j,n) and (j',n'). $b\left(\mathbf{p}^h\right)$ is a non-homothetic price aggregator which captures the relative price of high-income elastic goods. Goods for which $\beta_{(j,n)} > 0$ have positive income elasticity, while goods for which $\beta_{(j,n)} < 0$ have negative income elasticity. For AIDS to be a proper demand system, the following parametric restrictions need to be satisfied:[14]

$$\sum_j \sum_n \alpha^h_{(j,n)} = 1 \tag{6}$$

$$\sum_j \sum_n \beta_{(j,n)} = 0 \tag{7}$$

$$\sum_j \sum_n \gamma_{(j,n)(j',n')} = 0 \quad \forall (j',n') \tag{8}$$

$$\gamma_{(j,n)(j',n')} = \gamma_{(j',n')(j,n)} \quad \forall (j,n), (j',n') \tag{9}$$

[14]Under these constraints, the budget shares equations share the properties of a demand function, that is, they are homogeneous of degree 0 in prices and total expenditure, sum of budget shares add up to 1 and they satisfy the symmetry of the Slutsky matrix.

Applying Shephard's Lemma to the indirect utility function, I can derive the individual expenditure shares as follows:

$$\begin{aligned} s^z_{(j,n)} &= s_{(j,n)}\left(w_z, \mathbf{p}^h\right) \\ &= \alpha^h_{(j,n)} + \sum_{j'}\sum_{n'} \gamma_{(j,n)(j',n')} \ln p^h_{(j',n')} + \beta_{(j,n)} \ln\left(\frac{w_z}{a\left(\mathbf{p}^h\right)}\right) \end{aligned} \tag{10}$$

According to this equation, if a consumer has relatively low nominal wage, then she spends relatively more on low-income elastic goods. Under the AIDS, I can describe the market by the behavior of a representative consumer with the inequality-adjusted average nominal wage, $\tilde{w}^h = \bar{w}^h e^{\Sigma^h}$, which depends on the average nominal wage in country h, $\bar{w}^h$, and the Theil index, $\sum^h \equiv \mathbb{E}\left[\frac{w^h}{\bar{w}^h} \ln\left(\frac{w^h}{\bar{w}^h}\right)\right]$, a measure of inequality within a country. It is therefore straightforward to derive the aggregate expenditure shares in country h:

$$\begin{aligned} S^h_{(j,n)} &= s_{(j,n)}\left(\tilde{w}^h, \mathbf{p}^h\right) \\ &= \alpha^h_{(j,n)} + \sum_{j'}\sum_{n'} \gamma_{(j,n)(j',n')} \ln p^h_{(j',n')} + \beta_{(j,n)} \ln\left(\frac{\tilde{w}^h}{a\left(\mathbf{p}^h\right)}\right) \end{aligned} \tag{11}$$

Similarly, adjusted for the price level, $a\left(\mathbf{p}^h\right)$, if country h has higher inequality-adjusted average nominal wage, $\tilde{w}^h$, either because of higher average nominal wage or higher inequality, then it spends relatively more on high-income elastic goods.

It is convenient to rewrite the individual expenditure effect under the AIDS as:

$$\begin{aligned} \widehat{\psi_z} &= \sum_j \sum_n \left(s^z_{(j,n)} - S^h_{(j,n)}\right)\left(-\widehat{p^h_{(j,n)}}\right) \\ &= -\ln\left(\frac{w_z}{\tilde{w}^h}\right) \underbrace{\sum_j \sum_n \beta_{(j,n)} \widehat{p^h_{(j,n)}}}_{\widehat{b^h}} \end{aligned} \tag{12}$$

Intuitively, for an individual z who has lower nominal wage relative to the representative consumer in the country, if the price of a low-income elastic good goes down, she is better off and vice versa. Note that I do not have to observe each individual z's expenditure share on each good (j,n) in order to compute the change in her consumer price index.

Plugging in the above expression for $\widehat{\psi_z}$, I can write the local welfare change of individual z under the AIDS that corresponds to an infinitesimal change in prices and nominal wages as follows:

$$\widehat{u_z} \quad = \quad \widehat{E^h} - \ln\left(\frac{w_z}{\tilde{w}^h}\right)\widehat{b^h} + \widehat{w_z} \tag{13}$$

The global welfare change of individual z under the AIDS between an initial scenario under trade and a counterfactual scenario can be derived by integrating each component of the equation above:[15]

$$\underbrace{u_z^{tr\rightarrow cf}}_{\text{total effect}} \quad = \quad \underbrace{\left(\frac{E^h_{cf}}{E^h_{tr}}\right)}_{\text{agg. exp. effect}} \underbrace{\left(\frac{w_z^{tr}}{\tilde{w}^h_{tr}}\right)^{-\ln(b^h_{cf}/b^h_{tr})}}_{\text{ind. exp. effect}} \underbrace{\left(\frac{w_z^{cf}}{w_z^{tr}}\right)}_{\text{income effect}} \tag{14}$$

$$\frac{E^h_{cf}}{E^h_{tr}} \quad = \quad \prod_{(j,n)} \left(\frac{p^{h,tr}_{(j,n)}}{p^{h,cf}_{(j,n)}}\right)^{S^h_{(j,n)}} \tag{15}$$

$$-\ln\left(\frac{b^h_{cf}}{b^h_{tr}}\right) \quad = \quad -\sum_j \sum_n \beta_{(j,n)} \ln\left(\frac{p^{h,cf}_{(j,n)}}{p^{h,tr}_{(j,n)}}\right) \tag{16}$$

Note that $\frac{E^h_{cf}}{E^h_{tr}}$ and $-\ln(\frac{b^h_{cf}}{b^h_{tr}})$ are functions of the set of prices in the two scenarios, the aggregate expenditure shares that are observed in the data and a model parameter, $\beta_{(j,n)}$. If $u_z^{tr\rightarrow cf} < 1$, individual z is worse off after the change and vice versa.

2.4 Heterogeneous labor with comparative advantage across sectors

My supply-side specification allows for heterogeneous labor with comparative advantage across sectors so that different labor types sort into different sectors. As a result, price changes resulting from trade liberalization have a differential impact on their nominal wages. I use an assignment model of the labor market parametrized with a Fréchet distribution. In this environment, workers with different unobservable characteristics but identical observable characteristics may be allocated to different sectors in a competitive equilibrium.[16] In particular, an arbitrary worker z of type λ draws a vector of efficiency

[15] Please see Appendix 9.2 for the derivation of the aggregate and individual expenditure effects between an initial scenario under trade and a counterfactual scenario.

units across different sectors from a multivariate Fréchet distribution:[17]

$$\begin{aligned} G(\boldsymbol{\epsilon}(z);\lambda) &= \Pr\left[e(z;j) \le \epsilon(z;j)\ \forall j\right] \\ &= \exp\left\{-\left(\sum_j \epsilon(z;j)^{-\theta(\lambda)}\right)\right\} \end{aligned} \tag{17}$$

where $\theta(\lambda) > 1$ governs within-type dispersion of efficiency units. Worker z inelastically supplies $\epsilon(z;j)$ efficiency units of labor if she chooses to work in sector j.

Production requires only one factor, labor.[18][19][20] The production function in country h, sector j, using l efficiency units of labor type λ is:[21]

$$y^h(l;\lambda,j) = A^h(\lambda)T(\lambda,j)l \tag{18}$$

$A^h(\lambda)$ is the productivity of type λ workers in country h and $T(\lambda,j)$ is the productivity of type λ workers who choose to work in sector j. $A^h(\lambda)$ captures the absolute advantage of type λ workers in country h. $T(\lambda,j)$ captures the comparative advantage of type λ workers in sector j. Consider the partial equilibrium in which output prices, $\left\{p^h_{(j,h)}\right\}_{j\in\mathcal{J}}$,

[16] I assume that the labor market is perfectly competitive, that is, there is no friction. Dix-Carneiro (2014) finds that workers' median costs of switching sectors range from 1.4 to 2.7 times individual annual average wages, but these vary tremendously across individuals with different observable characteristics. For example, female, less educated, and older workers face substantially higher costs of switching as a fraction of individual wages. This increases the probability of unemployment of the low-skilled, and biases the gains from trade towards the high-skilled and high-income.

[17] Fréchet distributions of productivity shocks across factors have been imposed in the recent closed-economy models of Lagakos and Waugh (2013) and Hsieh et al. (2013) as well as the open economy models of Burstein, Morales and Vogel (2014), Costinot, Donaldson and Smith (2014), and Fajgelbaum and Redding (2014). Sector and country characteristics are assumed to be perfectly observed by the econometrician, but factor characteristics are not. See Costinot and Vogel (2015) for a detailed discussion.

[18] For simplicity, I abstract from capital in my production function. Capital may matter for two reasons. First, it may generate comparative advantage across sectors. This is very similar to introducing Hicks-neutral capital where capital is more important in some sectors than others. That would generate technological differences at the country-sector level. Capital reallocation reinforces labor reallocation in response to trade liberalization. Second, capital may be differentially complementary to different types of labor. In that case, there is a large number of cross elasticities I need to estimate, which is challenging.

[19] I do not feature complementarity between different types of equipment and heterogeneous workers across sectors as in Burstein, Morales and Vogel (2016) because I do not have data to compute the share of total hours worked by each labor group that is spent using different equipment types across sectors.

[20] My model is static, so I am not taking any stand on the accumulation of skills and capital in response to trade liberalization. I take each country's endowments of skills and capital as given for now, but it would be great to introduce dynamics to the framework.

[21] My model does not feature Ricardian-type country-sector productivity. However, I demonstrate in Appendix 9.3 that I am not understating the specialization of skill-abundant countries in skill-intensive sectors.

are given. Perfect competition and free entry entail that the per efficiency unit wage $x^h(\lambda, j)$ of a worker of labor type λ working in sector j in country h is:

$$x^h(\lambda, j) = p^h_{(j,h)} A^h(\lambda) T(\lambda, j) \tag{19}$$

Worker $z \in \mathcal{Z}^h(\lambda)$ with realization of the vector of efficiency units $\boldsymbol{\epsilon}(z) = \{\epsilon(z; j)\}_{j \in \mathcal{J}}$ needs to choose the sector that maximizes her labor earnings which is the product of her draw of efficiency units and per efficiency unit wage:

$$w_z = \max_j w_z(j) = \epsilon(z; j) \cdot x^h(\lambda, j) \tag{20}$$

The multivariate Fréchet distribution implies that the probability of a type λ worker choosing to work in sector j in country h is:

$$\begin{aligned} \pi^h(\lambda, j) &= \frac{\left[p^h_{(j,h)} A^h(\lambda) T(\lambda, j)\right]^{\theta(\lambda)}}{\sum_{j' \in \mathcal{J}} \left[p^h_{(j',h)} A^h(\lambda) T(\lambda, j')\right]^{\theta(\lambda)}} \\ &= \frac{x^h(\lambda, j)^{\theta(\lambda)}}{x^h(\lambda)^{\theta(\lambda)}} \end{aligned} \tag{21}$$

where $x^h(\lambda) \equiv \left(\sum_j x^h(\lambda, j)^{\theta(\lambda)}\right)^{\frac{1}{\theta(\lambda)}}$. With a higher $\theta(\lambda)$, which implies that there is less dispersion of efficiency units across sectors, a change in price or a change in productivity affects the factor allocation even more.

As a result, the worker sorting pattern is determined by comparative advantage:

$$\left[\frac{\pi^h(\lambda', j')}{\pi^h(\lambda', j)}\right]^{\frac{1}{\theta(\lambda')}} / \left[\frac{\pi^h(\lambda, j')}{\pi^h(\lambda, j)}\right]^{\frac{1}{\theta(\lambda)}} = \left[\frac{T(\lambda', j')}{T(\lambda', j)}\right] / \left[\frac{T(\lambda, j')}{T(\lambda, j)}\right] \tag{22}$$

If type λ' workers (relative to type λ workers) have a comparative advantage in sector j' (relative to sector j), then they are relatively more likely to sort into sector j', adjusted for potentially different values of $\theta(\lambda)$ and $\theta(\lambda')$. For larger $\theta(\lambda')$ (i.e. less dispersion in efficiency units among type λ' workers), it is even more likely for them to sort into sector j', in which they have a comparative advantage.

The distribution for $w_z = \max_j w_z(j)$ conditional on $z \in \mathcal{Z}^h(\lambda)$ is:

$$\Pr\left(w_z \le w \mid z \in \mathcal{Z}^h(\lambda)\right) = \exp\left\{-x^h(\lambda)^{\theta(\lambda)} w^{-\theta(\lambda)}\right\} \tag{23}$$

It is also distributed Fréchet with the scale parameter, $x^h(\lambda)$, the average per efficiency unit wage of labor type λ across the sectors, along with the dispersion parameter, $\theta(\lambda)$.[22]

The average nominal wage, $\bar{w}^h$, and the Theil index, $\sum^h$, in country h can also be expressed in terms of $x^h(\lambda)$ and $\theta(\lambda)$:

$$\bar{w}^h = \sum_{\lambda} \frac{L^h(\lambda)}{L^h} \Gamma(\lambda) x^h(\lambda) \tag{24}$$

$$\Sigma^h = \frac{1}{\bar{w}^h} \sum_{\lambda} \frac{L^h(\lambda)}{L^h} \Gamma(\lambda) \left(x^h(\lambda) \ln x^h(\lambda) - \frac{\Psi(\lambda)}{\theta(\lambda)} x^h(\lambda) \right) - \ln \bar{w}^h \tag{25}$$

where $\Gamma(\lambda) \equiv \Gamma\left(1 - \frac{1}{\theta(\lambda)}\right)$ is the gamma function and $\Psi(\lambda) \equiv \Psi\left(1 - \frac{1}{\theta(\lambda)}\right)$ is the digamma function.

2.5 General Equilibrium

In the general equilibrium, output prices, $\left\{p^h_{(j,h)}\right\}_{j \in \mathcal{J}}$, are determined by the market clearing conditions:

$$\sum_{\lambda} y^h\left(L^h(\lambda)\pi^h(\lambda, j); \lambda, j\right) = \sum_n \tau^n_{(j,h)} D^n_{(j,h)} \quad \forall j \in \mathcal{J} \tag{26}$$

where $y^h = A^h(\lambda)T(\lambda, j)\Gamma(\lambda)\pi^h(\lambda, j)^{1-\frac{1}{\theta(\lambda)}} L^h(\lambda)$ is the supply of sector j good by labor type λ in country h.[23] $\tau^n_{(j,h)}$ is the bilateral trade cost between export country h and import country n in sector j. $D^n_{(j,h)} = \left(S^n_{(j,h)} \bar{w}^n L^n \right) / p^n_{(j,h)}$ is country n's demand for good (j, h), where $S^n_{(j,h)}$ is given in equation (11) and $\bar{w}^n$ in equation (24). It depends on country n's wage distribution (in particular, $\bar{w}^n$ and Σ^n) as well as the vector of prices that consumers face in that country $\mathbf{p}^n$ (in particular, $p^n_{(j,h)} = \tau^n_{(j,h)} p^h_{(j,h)}$). Since these output prices enter both the demand side and the supply side nonlinearly, I apply the Gauss-Jacobi algorithm, an iterative method, to solve the system of market clearing equations numerically.[24] I also appeal to the Implicit Function Theorem to show that the price equilibrium that I have found numerically is locally isolated as a function of the parameters.[25] That is, in response to a small perturbation, if there exists an equilibrium,

[22] Burstein, et al. (2016) find that the wage distribution implied by the assumption of Fréchet distributions is a good approximation to the observed distribution of individual wages.

[23] Please see Appendix 9.4 for the derivation of the total supply.

[24] I demonstrate the existence of an equilibrium numerically.

[25] Please refer to Appendix 9.5 for a brief discussion of the Gauss-Jacobi Algorithm and the local property of the equilibrium.

then the system stays in the neighborhood of that equilibrium. I find no quantitative evidence of multiple equilibria.[26]

3 Analytical Results

In this section, I study these two channels and their interaction analytically. I consider a simple case where there are two countries, two sectors and two labor groups to illustrate the intuition.

Setup

Suppose that there are two countries, $h = 1, 2$, two sectors, $j = 1, 2$ and two labor groups, $\lambda = 1, 2$. The two labor groups differ in their skill levels, which allows nominal wages to vary across workers within a country. I assume that $\lambda = 1$ is high skilled. Goods produced in each sector are homogeneous and not differentiated by country of origin. As a result, there are two goods in total, $j = 1, 2$, which simplifies the analysis that follows. These two goods, however, have different income elasticity, which are $\beta > 0$ and $-\beta$, respectively.[27] That is, good 1 is high-income elastic relative to good 2. The implied non-homothetic preferences allow price indices to vary across consumers within a country. I further assume that good 1 is more skill intensive, based on the empirical finding that there is a positive correlation between the skill intensity of a good and its income elasticity. Allowing the two goods to differ in their skill intensity also leads to comparative advantage of different labor groups across sectors, which is necessary in generating the pattern of trade consistent with the Heckscher-Ohlin model. Finally, I assume perfect competition in all markets as before.

Demand

Suppose that country h's taste for good 1 independently from prices or income, $\alpha_1^h = \alpha$, and for good 2, $\alpha_2^h = 1 - \alpha$, $h = 1, 2$.[28] I further assume that the semi-elasticity of the

[26] I have tried multiple starting points and the system always converges to the same equilibrium. I have not proven either existence or uniqueness analytically. It is a complicated model with interactions and is not mapped neatly into the class of models considered in Alvarez and Lucas (2007).

[27] More precisely, β is semi-elasticity since it relates expenditure shares to log of income, but I refer to it as elasticity to save notation.

[28] One reason that $\alpha_1^1 \neq \alpha_1^2$ or $\alpha_2^1 \neq \alpha_2^2$ is because of home market effect. However, since I consider the case where both countries produce both goods, and that each good is homogeneous regardless of country of origin, it is reasonable to assume that $\alpha_1^1 = \alpha_1^2$ and $\alpha_2^1 = \alpha_2^2$.

expenditure share in one good with respect to the price of another, $\gamma_{12} = \gamma_{21} = \gamma$, and that with respect to its own price, $\gamma_{11} = \gamma_{22} = -\gamma$. Under these parametric restrictions, the budget shares equations in the Almost Ideal Demand System share the properties of a demand function. In addition, I assume that the outlay required for a minimal standard of living when prices are unity, $\underline{\alpha} = 0$, following the literature. The homothetic price aggregator becomes:

$$a(\mathbf{p}^h) = \exp\left\{\alpha \ln p_1^h + (1-\alpha)\ln p_2^h + \gamma \ln p_1^h \ln p_2^h\right\} = (p_1^h)^{\alpha}(p_2^h)^{1-\alpha}(p_2^h)^{\gamma \ln p_1^h} \tag{27}$$

The non-homothetic price aggregator becomes:

$$b(\mathbf{p}^h) = \exp\left\{\beta \ln p_1^h - \beta \ln p_2^h\right\} = \left(\frac{p_1^h}{p_2^h}\right)^{\beta} \tag{28}$$

The aggregate expenditure shares in country h are:

$$S_1^h = \alpha - (\gamma + \alpha\beta)\ln p_1^h + \left[\gamma - \beta(1-\alpha)\right]\ln p_2^h - \beta\gamma \ln p_1^h \ln p_2^h + \beta \ln \tilde{w}^h \tag{29}$$

$$S_2^h = 1 - \alpha + (\gamma + \alpha\beta)\ln p_1^h - \left[\gamma - \beta(1-\alpha)\right]\ln p_2^h + \beta\gamma \ln p_1^h \ln p_2^h - \beta \ln \tilde{w}^h \tag{30}$$

Country h is endowed with H^h high-skilled workers and L^h low-skilled workers, and they are paid w_H^h and w_L^h, respectively. Suppose the two countries are of the same size, that is, $H^1 + L^1 = H^2 + L^2 = N$, but country 1 is more skill abundant, $\frac{H^1}{L^1} > \frac{H^2}{L^2}$. The inequality-adjusted average nominal wage, $\tilde{w}^h = \bar{w}^h e^{\sum^h}$, where $\bar{w}^h = \frac{1}{N}(H^h w_H^h + L^h w_L^h)$. Therefore,

$$\ln \tilde{w}^h = \ln \bar{w}^h + \sum^h = \ln \bar{w}^h + \frac{1}{N}\left[H^h \frac{w_H^h}{\bar{w}^h}\ln\left(\frac{w_H^h}{\bar{w}^h}\right) + L^h \frac{w_L^h}{\bar{w}^h}\ln\left(\frac{w_L^h}{\bar{w}^h}\right)\right] \tag{31}$$

Production

The production side follows closely the Heckscher-Ohlin model with Cobb-Douglas production function:

$$y_j^h = (H_j^h)^{\eta_j}(L_j^h)^{1-\eta_j}, \; j, h = 1, 2 \tag{32}$$

where $0 < \eta_2 < \eta_1 < 1$.

A producer in country h, sector j, solves the following cost minimization problem:

$$\begin{aligned} & \min w_H^h H_j^h + w_L^h L_j^h \\ & s.t.\ y_j^h \leq (H_j^h)^{\eta_j} (L_j^h)^{1-\eta_j} \end{aligned} \tag{33}$$

This implies the following equation, under the assumption that both countries produce both goods:

$$\frac{\eta_j}{1-\eta_j} L_j^h w_L^h \;=\; H_j^h w_H^h \tag{34}$$

Total supply of good j produced by country h is, therefore, $L_j^h \left(\frac{\eta_j}{1-\eta_j} \frac{w_L^h}{w_H^h} \right)^{\eta_j}$. Zero profits, $p_j^h (H_j^h)^{\eta_j} (L_j^h)^{1-\eta_j} = w_H^h H_j^h + w_L^h L_j^h$, combined with equation (34), lead to:

$$p_j^h \;=\; \frac{(w_L^h)^{1-\eta_j} (w_H^h)^{\eta_j}}{(1-\eta_j)^{1-\eta_j} \eta_j^{\eta_j}} \tag{35}$$

Autarky Equilibrium

The goods market clearing condition implies that:

$$S_j^h \bar{w}^h N / p_j^h \;=\; L_j^h \left(\frac{\eta_j}{1-\eta_j} \frac{w_L^h}{w_H^h} \right)^{\eta_j} \tag{36}$$

Substitute p_j^h using equation (35),

$$w_L^h L_j^h \;=\; S_j^h \bar{w}^h N \left(1 - \eta_j\right) \tag{37}$$

Use good 2 as the numeraire, that is, $p_2^h = 1$, $h = 1, 2$, then equations (29) and (30) become:

$$S_1^h \;=\; \alpha - (\gamma + \alpha\beta) \ln p_1^h + \beta \ln \tilde{w}^h \tag{38}$$

$$S_2^h \;=\; 1 - \alpha + (\gamma + \alpha\beta) \ln p_1^h - \beta \ln \tilde{w}^h \tag{39}$$

In addition, according to equation (35), $p_1^h = \frac{(w_L^h)^{1-\eta_1} (w_H^h)^{\eta_1}}{(1-\eta_1)^{1-\eta_1} \eta_1^{\eta_1}}$ and $p_2^h = \frac{(w_L^h)^{1-\eta_2} (w_H^h)^{\eta_2}}{(1-\eta_2)^{1-\eta_2} \eta_2^{\eta_2}} = 1$. The latter implies that:

$$w_L^h \;=\; (1-\eta_2) \eta_2^{\frac{\eta_2}{1-\eta_2}} (w_H^h)^{\frac{-\eta_2}{1-\eta_2}} \tag{40}$$

Substitute w_L^h using equation (40),

$$p_1^h = \frac{(1-\eta_2)^{1-\eta_1}\eta_2^{\frac{\eta_2(1-\eta_1)}{1-\eta_2}}}{(1-\eta_1)^{1-\eta_1}\eta_1^{\eta_1}}(w_H^h)^{\frac{1+\eta_1\eta_2-2\eta_2}{1-\eta_2}} \tag{41}$$

Furthermore, equation (37) implies that:

$$w_L^h L_1^h = S_1^h \bar{w}^h N (1-\eta_1) \tag{42}$$
$$w_L^h L_2^h = S_2^h \bar{w}^h N (1-\eta_2) \tag{43}$$

which add up to the following equation:

$$w_L^h L^h = \bar{w}^h N \left[(1-\eta_1)S_1^h + (1-\eta_2)S_2^h\right] \tag{44}$$

Since $S_1^h = 1 - S_2^h$, I simplify equation (44) further and obtain:

$$w_L^h L^h = \bar{w}^h N \left[1-\eta_2 + (\eta_2-\eta_1)S_1^h\right] \tag{45}$$

Note that every variable in this equation is a function of w_H^h and model parameters, and once I solve for w_H^h, I can back out p_1^h, w_L^h, L_1^h and L_2^h. Finally, using equation (34), I can back out H_1^h and H_2^h as well. These variables characterize the autarky equilibrium of the two countries.

Case 1: $\beta = 0$

Under this restriction, the demand system is homothetic. For simplicity, I further impose that $\gamma = 0$, that is, there is no substitution across goods. As a result, $S_1^h = \alpha$ and $S_2^h = 1-\alpha$. Substitute w_L^h using equation (40),

$$w_H^h = \left\{\frac{\left[(1-\eta_2)+(\eta_2-\eta_1)\alpha\right]H^h}{\left[\eta_2+(\eta_1-\eta_2)\alpha\right]L^h}\right\}^{\eta_2-1} \tag{46}$$

Case 2: $\eta_1 = \eta_2 = \frac{1}{2}$

Under this restriction, equation (40) implies that $w_L^h = \frac{1}{4w_H^h}$. Equation (45) then implies that:

$$\frac{1}{4w_H^h} = \frac{(1-\eta_2)}{L^h} N \left[\frac{H^h}{N} w_H^h + \frac{L^h}{N} \frac{1}{4w_H^h} \right] \tag{47}$$

Solve the equation,

$$w_H^h = \frac{1}{2}\sqrt{\frac{L^h}{H^h}} \tag{48}$$

Free Trade Equilibrium

In the two country example, free trade implies that the price of each good is the same in both countries, that is, $p_1^1 = p_1^2 = p_1$ and $p_2^1 = p_2^2 = p_2$. I consider the case in which both countries produce both goods. The producer cost minimization problem implies, as in equation (34), that $\frac{\eta_j L_j^h}{1-\eta_j} \frac{w_L^h}{w_H^h} = H_j^h$. Combined with zero profits,

$$p_j = \frac{(w_L^h)^{1-\eta_j} (w_H^h)^{\eta_j}}{(1-\eta_j)^{1-\eta_j} \eta_j^{\eta_j}}, \; j, h = 1, 2 \tag{49}$$

Since technologies and prices are the same in the two countries, equation (49) implies that:

$$\begin{aligned} (w_L^1)^{1-\eta_1} (w_H^1)^{\eta_1} &= (w_L^2)^{1-\eta_1} (w_H^2)^{\eta_1} \\ (w_L^1)^{1-\eta_2} (w_H^1)^{\eta_2} &= (w_L^2)^{1-\eta_2} (w_H^2)^{\eta_2} \end{aligned} \tag{50}$$

Replace these two equations in one another: $w_H^1 = w_H^2 = w_H$ and $w_L^1 = w_L^2 = w_L$. That is, factor price equalization (FPE) holds in the free trade equilibrium.

From the cost minimization of the producer:

$$p_j y_j^h = \frac{w_H H_j^h}{\eta_j} \tag{51}$$

Sum up over h and use FPE:

$$p_j (y_j^1 + y_j^2) = \frac{w_H}{\eta_j} (H_j^1 + H_j^2) \tag{52}$$

Use goods market clearing condition:

$$S_j^1(H^1 w_H + L^1 w_L) + S_j^2(H^2 w_H + L^2 w_L) = p_j(y_j^1 + y_j^2) = \frac{w_H}{\eta_j}(H_j^1 + H_j^2) \tag{53}$$

Add up over j:

$$\begin{aligned}\sum_j \eta_j \Bigg[S_j^1(H^1 w_H + L^1 w_L) + S_j^2(H^2 w_H + L^2 w_L)\Bigg] &= w_H \sum_j (H_j^1 + H_j^2) \\ &= w_H(H^1 + H^2) \end{aligned} \tag{54}$$

Since $S_2^1 = 1 - S_1^1$ and $S_2^2 = 1 - S_1^2$, equation (54) can be rewritten as:

$$(H^1 w_H + L^1 w_L)\Bigg[(\eta_1 - \eta_2)S_1^1 + \eta_2\Bigg] + (H^2 w_H + L^2 w_L)\Bigg[(\eta_1 - \eta_2)S_1^2 + \eta_2\Bigg] = w_H(H^1 + H^2) \tag{55}$$

where

$$S_1^h = \alpha - (\gamma + \alpha\beta)\ln p_1 + \beta \ln \tilde{w}^h \tag{56}$$

$$S_2^h = 1 - \alpha + (\gamma + \alpha\beta)\ln p_1 - \beta \ln \tilde{w}^h \tag{57}$$

$$p_1 = \frac{(w_L)^{1-\eta_1}(w_H)^{\eta_1}}{(1-\eta_1)^{1-\eta_1}\eta_1^{\eta_1}} \tag{58}$$

$$p_2 = \frac{(w_L)^{1-\eta_2}(w_H)^{\eta_2}}{(1-\eta_2)^{1-\eta_2}\eta_2^{\eta_2}} \tag{59}$$

Use good 2 as the numeraire, that is, $p_2 = 1$, I obtain:

$$w_L = (1-\eta_2)\eta_2^{\frac{\eta_2}{1-\eta_2}}(w_H)^{\frac{-\eta_2}{1-\eta_2}} \tag{60}$$

and it follows that:

$$p_1 = \frac{(1-\eta_2)^{1-\eta_1}\eta_2^{\frac{\eta_2(1-\eta_1)}{1-\eta_2}}}{(1-\eta_1)^{1-\eta_1}\eta_1^{\eta_1}}(w_H)^{\frac{1+\eta_1\eta_2-2\eta_2}{1-\eta_2}} \tag{61}$$

Note that every variable in equation (55) is a function of w_H and model parameters, and once I solve for w_H, I can back out p_1 and w_L.

Recall that the capital-labor ratio implies that:

$$\frac{\eta_j L_j^h w_L}{(1-\eta_j) w_H} = H_j^h \tag{62}$$

Add that up for both goods:

$$\frac{w_L}{w_H}\left[\frac{\eta_1}{1-\eta_1}(L^h - L_2^h) + \frac{\eta_2}{1-\eta_2} L_2^h\right] = H^h \tag{63}$$

Equation (63) can be rearranged as the following:

$$\frac{L_2^h}{L^h} = \frac{(1-\eta_2)(1-\eta_1)}{\eta_2 - \eta_1}\left[\frac{w_H H^h}{w_L L^h} - \frac{\eta_1}{1-\eta_1}\right] \tag{64}$$

which can be used to solve for L_2^h and L_1^h, $h = 1, 2$. Finally, using equation (62), I can back out H_1^h and H_2^h as well. These variables characterize the free trade equilibrium of the world economy.

Case 1: $\beta = 0$

Under this restriction, the demand system is homothetic. For simplicity, I further impose that $\gamma = 0$, that is, there is no substitution across goods. As a result, $S_1^h = \alpha$ and $S_2^h = 1 - \alpha$. Substitute w_L^h using equation (60) into equation (55):

$$w_H^h = \left\{\frac{\Big[(1-\eta_2) + (\eta_2 - \eta_1)\alpha\Big]}{\Big[\eta_2 + (\eta_1 - \eta_2)\alpha\Big]}\left(\frac{H^1 + H^2}{L^1 + L^2}\right)\right\}^{\eta_2 - 1} \tag{65}$$

Case 2: $\eta_1 = \eta_2 = \frac{1}{2}$

Under this restriction, equation (55) implies that:

$$\eta_2(H^1 w_H + L^1 w_L + H^2 w_H + L^2 w_L) = w_H(H^1 + H^2) \tag{66}$$

Substitute $w_L^h = \frac{1}{4 w_H^h}$ into the above equation:

$$w_H^h = \frac{1}{2}\sqrt{\frac{L^1 + L^2}{H^1 + H^2}} \tag{67}$$

Compare the two cases in the autarky equilibrium with the free trade equilibrium, I find that suppose $\frac{H^1}{L^1} > \frac{H^2}{L^2}$,

$$\begin{cases} w_H^{aut} < w_H^{ft} & for\, h = 1 \\ w_H^{aut} > w_H^{ft} & for\, h = 2 \end{cases}$$
$$\begin{cases} w_L^{aut} > w_L^{ft} & for\, h = 1 \\ w_L^{aut} < w_L^{ft} & for\, h = 2 \end{cases} \tag{68}$$

which is consistent with the prediction based on the Heckscher-Ohlin model, but inconsistent with my quantitative result in the next section that the relative nominal wage of the skilled workers in skill-scarce countries also goes up after trade liberalization.

Recall that the goods market clearing condition in the autarky equilibrium is:

$$w_L^h L^h = \bar{w}^h N \left\{ 1 - \eta_2 + (\eta_2 - \eta_1) \left[\alpha - (\gamma + \alpha\beta) \ln p_1^h + \beta \ln \tilde{w}^h \right] \right\} \tag{69}$$

and the goods market clearing condition in the free trade equilibrium is:

$$\begin{aligned} w_H(H^1 + H^2) = & \; (w_H H^1 + w_L L^1) \left\{ \eta_2 + (\eta_1 - \eta_2) \left[\alpha - (\gamma + \alpha\beta) \ln p_1 + \beta \ln \tilde{w}^1 \right] \right\} + \\ & (w_H H^2 + w_L L^2) \left\{ \eta_2 + (\eta_1 - \eta_2) \left[\alpha - (\gamma + \alpha\beta) \ln p_1 + \beta \ln \tilde{w}^2 \right] \right\} \end{aligned} \tag{70}$$

Trade liberalization in the form of lower trade costs increases $\ln \tilde{w}^1$ and $\ln \tilde{w}^2$ exogenously. In both cases, it is only when $\beta \neq 0$ and $\eta_1 \neq \eta_2$ that this increase leads to a rise in the relative nominal wage of the skilled workers in both countries according to equations (69) and (70) since $\eta_1 > \eta_2$. I also solve the two equilibria numerically and find consistent result with my quantitative exercise for reasonable parameter values.

4 Data

For the demand-side estimation, I use mainly the World Input-Output Database (WIOD), which provides information on bilateral trade flows and production data for 40 countries (27 European countries and 13 other large countries) and 35 sectors in the economy. It also distinguishes between final consumption and intermediate uses.[29]

World Input-Output Table looks like the following:

			Use by country-industries							Final use by countries			Total use
			Country 1			...	Country M			Country 1	...	Country M	
			Industry 1	...	Industry N	...	Industry 1	...	Industry N		...		
Supply from country-industries	Country 1	Industry 1											
		...											
		Industry N											
	......												
	Country M	Industry 1											
		...											
		Industry N											
Value added by labour and capital													
Gross output													

Figure 1: Schematic Outline of a World Input-Output Table (WIOT)

For the supply-side estimation, I use mainly the Integrated Public Use Microdata Series, International (IPUMS-I), which provides publicly available nationally representative survey data for 82 countries that are coded and documented consistently across countries and over time. It also provides individual-level data with labor incomes and worker characteristics. I divide the workers in IPUMS-I dataset into 18 disjoint groups, Λ, by age (15-24, 25-49 and 50-74), gender (male and female) and educational attainment (ED0-2, less than primary, primary and lower secondary education; ED3-4, upper secondary and post-secondary non-tertiary education; ED5-8, tertiary education).

5 Parametrization

5.1 Supply-side Parameters

On the supply side, I need to estimate $\theta(\lambda)$, the worker type specific Fréchet dispersion parameter, $L^h(\lambda)/L^h$, the fraction of type λ workers in country h, $A^h(\lambda)$, the productivity of type λ workers in country h and $T(\lambda, j)$, the productivity of type λ workers who choose to work in sector j.

To estimate the worker type specific Fréchet dispersion parameter $\theta(\lambda)$, I follow the methodology in Lagakos and Waugh (2013) and Hsieh et al. (2013) and match the moments of the empirical distribution of within type worker wages.[30] In particular, the mean and the variance of nominal wages within a labor group satisfy:

[29] I do not use the UN Comtrade Database because it does not have information on the input-output transactions of a country with itself.

$$\frac{VAR\left[w_z \mid z \in \mathcal{Z}^h(\lambda)\right]}{E\left[w_z \mid z \in \mathcal{Z}^h(\lambda)\right]^2} = \frac{\Gamma\left(1 - \frac{2}{\theta(\lambda)}\right)}{\Gamma\left(1 - \frac{1}{\theta(\lambda)}\right)^2} - 1 \tag{71}$$

I restrict my sample in the following way: I drop workers who are younger than 15 years old, are self-employed or work part-time (<30 hours per week), do not report positive labor earnings, or have missing information on age, sex or education. I also drop the top and bottom 1% of earners to remove potential outliers, and to minimize the impact of potential cross-country differences in top-coding procedures. All calculations in my analysis are weighted using the applicable sample weights. I measure w_z as the annual labor earnings; $\epsilon(z; j)$ captures both the hours worked and efficiency units of worker z who chooses to work in sector j; $\theta(\lambda)$ reflects dispersion in both hours worked and efficiency units of type λ workers; $L^h(\lambda)$ is the headcount of type λ workers.

I use IPUMS-I to estimate $\theta(\lambda)$ for 16 countries.[31] Since the estimates of $\theta(\lambda)$ are very close across the 16 countries for each labor type λ, I use the average of these estimates for all countries and assume that $\theta(\lambda)$ doesn't change over time. I back out $x^h(\lambda)$ using $\mathbb{E}\left[w_z \mid z \in \mathcal{Z}^h(\lambda)\right] = x^h(\lambda)\Gamma(1 - \frac{1}{\theta(\lambda)})$ for the 16 countries. Since all earnings data in IPUMS-I are in local currency units, I use the official exchange rate (LCU per US$, period average) from the World Bank to convert all values to US$. I also find that output-side real GDP per capita have strong explanatory power for $x^h(\lambda)$, so I use the predicted values of $x^h(\lambda)$ for the rest of the countries.[32]

Since IPUMS-I does not provide information on $L^h(\lambda)/L^h$ for all of the 40 countries, I use the following complementary datasets. First, I use Eurostat, which provides information on the full-time and part-time employment by age, gender and educational attainment. It includes 27 European countries in WIOD. Second, I use UNdata, which has information on population 15 years of age and over, also by age, gender and educational attainment, for Russia, Australia, Korea and China. This dataset comes from UNSD Demographic Statistics–United Nations Statistics Division. Third, I use National Statistics, Republic of China (Taiwan) and finally, Population Statistics of Japan.

In order to estimate the sector-level non-homothetic gravity equation, which I explain

[30] As a robustness check, I also jointly estimate $\theta(\lambda)$ and $x^h(\lambda)$ for each labor type using maximum likelihood.

[31] The list of countries can be found in Appendix 9.6.

[32] I get the data on output-side real GDP at chained PPPs (in millions of 2005 U.S.$) and population from the Penn World Tables.

in detail in the next section, I need to compute the inequality-adjusted average nominal wage of each country, which requires an estimate of its average nominal wage as well as its Theil index. Table 1 reports my estimates of the average labor earnings and the Theil index for the 40 countries based on equations (24) and (25). I estimate $\bar{w}^h$ and $\sum^h$ for the years 2005, 2006 and 2007, and then take the average.

Country	Theil	Avg Labor Earnings	Country	Theil	Avg Labor Earnings
AUS	0.17	35871	IRL	0.18	45164
AUT	0.18	31585	ITA	0.17	25381
BEL	0.17	31446	JPN	0.17	30438
BGR	0.19	7196	KOR	0.18	23422
BRA	0.32	2835	LTU	0.17	11927
CAN	0.17	37134	LUX	0.17	60919
CHN	0.34	1661	LVA	0.18	9889
CYP	0.18	17773	MEX	0.23	3813
CZE	0.17	18342	MLT	0.20	13412
DEU	0.16	33901	NLD	0.17	39566
DNK	0.17	34748	POL	0.17	11096
ESP	0.19	25098	PRT	0.19	14326
EST	0.17	14544	ROU	0.19	6365
FIN	0.17	32274	RUS	0.18	11210
FRA	0.18	27794	SVK	0.17	12936
GBR	0.18	31318	SVN	0.17	19767
GRC	0.18	20335	SWE	0.17	33596
HUN	0.17	12821	TUR	0.21	6884
IDN	0.20	1378	TWN	0.21	21729
IND	0.40	737	USA	0.19	41898

Table 1: Average Labor Earnings and the Theil Index

Recall that the Theil index measures the level of inequality within a country, which in my framework is the dispersion in labor incomes. Since my Theil indices are calculated using only the labor earnings of the population aged between 15 and 74, I also use IPUMS-I to construct alternative measures of wage Gini coefficients using three different methods that are widely used in the literature. Let y_i be the labor income of a person indexed in non-decreasing order ($y_i \leq y_{i+1}$), my first two measures of the wage Gini coefficients are calculated as follows: $G_1 = \frac{2\sum_{i=1}^{n} iy_i}{n\sum_{i=1}^{n} y_i} - \frac{n+1}{n}$ and $G_2 = 1 - \frac{2}{n-1}(n - \frac{\sum_{i=1}^{n} iy_i}{\sum_{i=1}^{n} y_i})$. On the other hand, Guillermina Jasso and Angus Deaton independently propose the following formula: $G_3 = \frac{N+1}{N-1} - \frac{2}{N(N-1)\mu}(\sum_{i=1}^{n} P_i X_i)$ where μ is mean income of the population, P_i is the income rank P of person i, with income, X_i, such that the richest person receives a rank

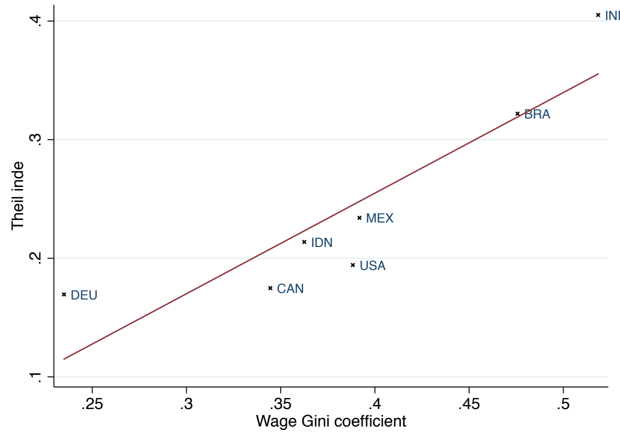

Figure 2: Wage Gini coefficient calculated using IPUMS-I

of 1 and the poorest a rank of N. The three methods give me very similar estimates and Figure 2 demonstrates that my model-implied Theil indices perform very well against the Jasso and Deaton measure. Their correlation is significantly positive at 0.89.

I plot in Figure 3 my model-implied Theil indices for all of the 40 countries against the Gini coefficients reported in the World Income Inequality Database that are computed using all sources of income. The two measures are still positively correlated and the correlation coefficient is 0.61. In the right panel, I exclude the three potential outliers and the correlation coefficient remains positive and is around 0.55.

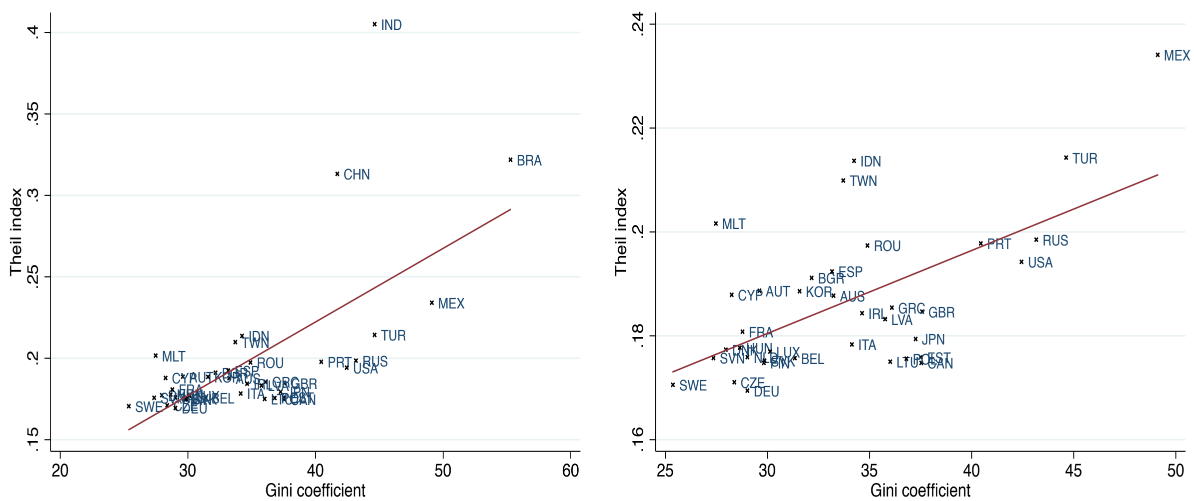

Figure 3: Theil Index

In Figure 4, I plot my model-implied labor earnings per capita against output-side GDP per capita. My measure of income per capita tracks the data very well. These parameter implications provide evidence that my model assumptions on the supply side do well at matching the data.

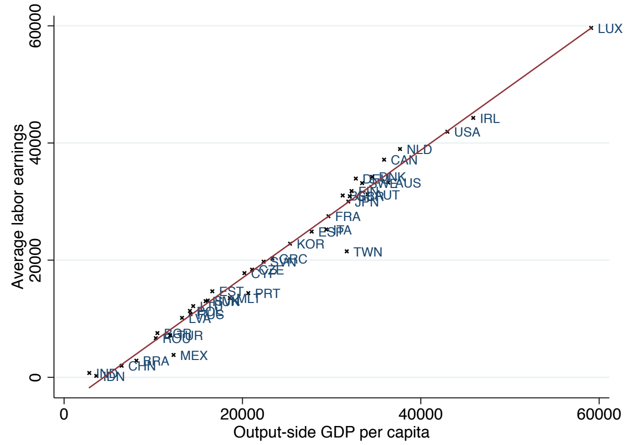

Figure 4: Average Labor Earnings

As discussed above, the worker sorting pattern can be used to parametrize $T(\lambda, j)$. I need $\Lambda + J - 1 = 52$ normalization. I pick $\lambda = 1$ such that $T(\lambda, j') = 1\ \forall j'$. I also pick $j = 1$ such that $T(\lambda', j) = 1\ \forall \lambda'$. Then I have $T(\lambda', j') = \left[\frac{\pi^h(\lambda',j')}{\pi^h(\lambda',j=1)}\right]^{\frac{1}{\theta(\lambda')}} / \left[\frac{\pi^h(\lambda=1,j')}{\pi^h(\lambda=1,j=1)}\right]^{\frac{1}{\theta(\lambda=1)}}$, $\forall \lambda' \neq 1\ \forall j' \neq 1$ where $\pi^h(\lambda, j) = L^h(\lambda, j)/L^h(\lambda)$ and $L^h(\lambda, j)$ is the headcount of type λ workers in country h that choose to work in sector j. Since there is no information on $L^h(\lambda, j)$ in Eurostat or UNdata, I use data on the countries that are available in IPUMS-I to compute $T(\lambda, j)$ and then use the average of the estimates for all of the countries.[33] Given the specified normalization,

$$\frac{\pi(\lambda', j)}{\pi(\lambda, j)} = \frac{T(\lambda', j)^{\theta(\lambda')}}{T(\lambda, j)^{\theta(\lambda)}} \tag{72}$$

I plot in Figure 5 this ratio aggregating the 18 labor groups into three broad categories based on educational attainment against an estimate of the skill intensity of each sector which matches the share of hours worked in that sector by workers with a completed

[33]This restriction implies, for example, a U.S. and a Chinese female worker who are 25-year-old and college educated are both twice as productive in health care than in mining. Because of data limitations, I cannot estimate $T(\lambda, j)$ for every country. This restriction is reasonable and does well in capturing the systematic relationship between the different labor types and the sectors that they sort into.

tertiary degree in the U.S.[34][35] The correlation coefficients are -0.41 and -0.52 respectively. These graphs illustrate that workers with less education are more likely to work in unskill-intensive sectors. This implies that a decline in the relative price of goods in unskill-intensive sectors decreases the relative nominal wage of unskilled workers.[36]

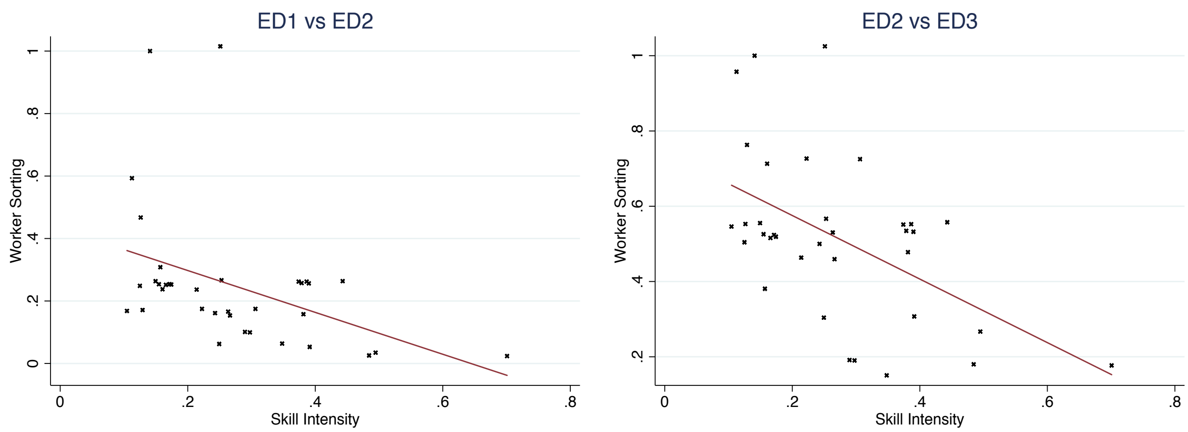

Figure 5: Worker Sorting

To estimate $A^h(\lambda)$, the productivity of type λ workers in country h, I take a first-order approximation of the following equation at $\boldsymbol{p} = \boldsymbol{1}$, $\boldsymbol{T} = \boldsymbol{1}$:

$$\begin{aligned} x^h(\lambda) = \left(\sum_j x^h(\lambda, j)^{\theta(\lambda)} \right)^{\frac{1}{\theta(\lambda)}} &= \left\{ \sum_{j \in \mathcal{J}} \left[p^h_{(j,h)} A^h(\lambda) T(\lambda, j) \right]^{\theta(\lambda)} \right\}^{\frac{1}{\theta(\lambda)}} \\ &= A^h(\lambda) \left\{ \sum_{j \in \mathcal{J}} \left[p^h_{(j,h)} T(\lambda, j) \right]^{\theta(\lambda)} \right\}^{\frac{1}{\theta(\lambda)}} \end{aligned} \tag{73}$$

which gives me:[37]

$$log\left(\frac{x^h(\lambda)}{x^h(1)}\right) = log\left(\frac{A^h(\lambda)}{A^h(1)}\right) + log\, J\left(\frac{1}{\theta(\lambda)} - \frac{1}{\theta(1)}\right) + \frac{1}{J}\sum_{j \in \mathcal{J}} log\left(\frac{T(\lambda, j)}{T(1, j)}\right) \tag{74}$$

[34] ED1 corresponds to less than primary, primary and lower secondary education; ED2 corresponds to upper secondary and post-secondary non-tertiary education; ED3 corresponds to tertiary education.

[35] I thank Jonathan Vogel for providing me with these estimates.

[36] In partial equilibrium, changes in wages are proportional to changes in output prices, where the weight depends on factor allocation in the initial period. An increase in sector j's output price raises the relative wage of labor groups that disproportionately work in sector j in the initial trade equilibrium.

[37] Please see Appendix 9.7 for the derivation of equation (30).

I assume that $A^h(\lambda = 1) = 1 \, \forall h$.[38] Figure 6 is a bar chart that plots the average $A^h(\lambda)$ across countries for each of the 18 labor goups by age, gender and educational attainment. As expected, for those who are of the same age and gender, the less education one receives, the lower the average estimate of $A^h(\lambda)$. In addition, for those who are of the same gender and have the same level of education, the younger one is, the lower the average estimate of $A^h(\lambda)$. Finally, a female worker is estimated to have lower average $A^h(\lambda)$ than her male counterpart. Zooming in on education, I aggregate the 18 labor groups into three broad categories. The bar chart on the right illustrates that less educated individuals have lower $A^h(\lambda)$ on average regardless of their age and gender. This implies that less educated workers have lower nominal wages regardless of their sectoral choices.

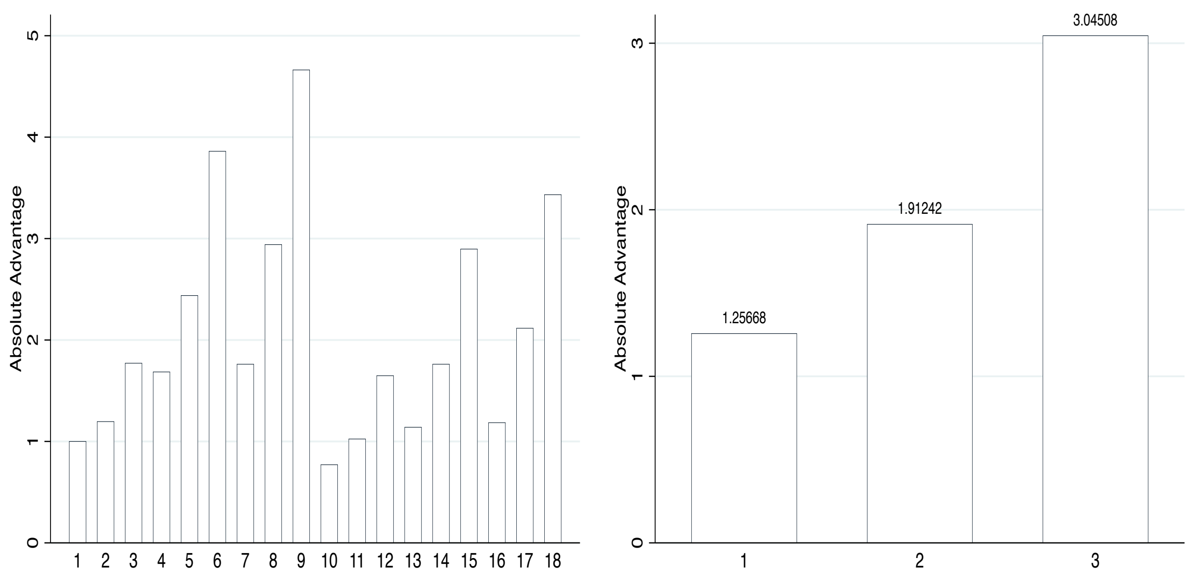

Figure 6: $A^h(\lambda)$ and education

5.2 Demand-side parameters

On the demand side, I need to estimate $\underline{\alpha}$, which can be interpreted as the outlay required for a minimal standard of living when prices are unity. I assign 0 to $\underline{\alpha}$ a priori. I also need to estimate the vector of income elasticities, $\boldsymbol{\beta} = \left\{\beta_{(j,n)}\right\}$, and the matrix of cross elasticities, $\boldsymbol{\Gamma} = \left\{\gamma_{(j,n)(j',n')}\right\}$, as well as $\alpha^h_{(j,n)}$, the overall taste in country h for the goods exported by country n in sector j independently from prices or income of the importer.

On top of the regularity restrictions imposed by the AIDS, I impose additional assumptions

[38] Please refer to Appendix 9.8 for a description of the characteristics of each labor group.

on the matrix $\mathbf{\Gamma}$ to reduce the number of parameters I estimate:

$$\gamma_{(j,n)(j',n')} = \begin{cases} \frac{\gamma_j}{N} & j = j', n \neq n' \\ -\left(1 - \frac{1}{N}\right)\gamma_j & j = j', n = n' \\ 0 & j \neq j' \end{cases} \tag{75}$$

In words, this implies that within the same sector, cross elasticities are the same between goods produced by different countries and across sectors, there is no substitution.[39]

Under these parametric restrictions, the sectoral non-homothetic gravity equation is:[40]

$$S^h_{(j,n)} \equiv \frac{Y^h_{(j,n)}}{Y^h} = \frac{Y_{(j,n)}}{Y^W} + K^h_{(j,n)} - \gamma_j M^h_{(j,n)} + \beta_{(j,n)}\Omega^h \tag{76}$$

where $\frac{Y_{(j,n)}}{Y^W}$ captures the size of the exporter n in sector j in the world economy; $K^h_{(j,n)} = \alpha^h_{(j,n)} - \sum_{n'}\left(\frac{Y^{n'}}{Y^W}\right)\alpha^{n'}_{(j,n)}$ captures the differences in taste across countries for different goods; $M^h_n = \ln\left(\frac{\tau^h_{(j,n)}}{\bar{\tau}_j{}^h}\right) - \sum_{n'}\left(\frac{Y^{n'}}{Y^W}\right)\ln\left(\frac{\tau^{n'}_{(j,n)}}{\bar{\tau}^{n'}_j}\right)$ captures bilateral trade costs and multilateral resistance, and $\Omega^h = y^h - \sum_{n'}\left(\frac{Y^{n'}}{Y^W}\right)y^{n'}$ is the non-homothetic component of the gravity equation. For example, a country with a high Ω^h, either because of its high average nominal wage or its high inequality, is predicted to consume more of the high-income elastic goods.

Following Fajgelbaum and Khandelwal (2016), I proxy $K^h_{(j,n)}$ with the product of the exporter fixed effect and country h's expenditure share on sector j relative to the world. Since I do not observe directly the trade costs between country pairs, I proxy them with bilateral observables.

To be more specific, I assume importer h's taste for good (j,n), $\alpha^h_{(j,n)}$, can be decomposed into an exporter effect, a_n, a sector effect, a_j, and an importer taste for that sector, ε^h_j:

$$\alpha^h_{(j,n)} \quad = \quad a_n\left(a_j + \varepsilon^h_j\right) \tag{77}$$

Under the additional assumptions on $\mathbf{\Gamma}$, aggregate expenditure shares are:

$$S^h_{(j,n)} = \alpha^h_{(j,n)} - \gamma_j \ln p^h_{(j,n)} + \frac{\gamma_j}{N}\sum_{n'=1}^{N}\ln p^h_{(j,n')} + \beta_{(j,n)}y^h \tag{78}$$

Therefore, the sectoral expenditure shares become:

[39] Normalization by the number of countries N is mainly for notational simplicity but not necessary.

[40] Please see Appendix 9.9 for the derivation of the sector-level non-homothetic gravity equation.

$$S_j^h = \sum_n S_{(j,n)}^h = \bar{\alpha}_j^h + \bar{\beta}_j y^h \tag{79}$$

where $\bar{\alpha}_j^h = \sum_n \alpha_{(j,n)}^h$ and $\bar{\beta}_j = \sum_n \beta_{(j,n)}$. In the absence of non-homotheticity, $\bar{\beta}_j = 0\,\forall j$. In that case, the upper tier is Cobb-Douglas with fixed expenditure shares $\left\{\bar{\alpha}_j^h\right\}_{j\in\mathcal{J}}$. I further impose the restriction: $\sum_{n=1}^N \alpha_n = 1$. This re-expresses $K_{(j,n)}^h = a_n\left(S_j^h - S_j^W\right) - a_n\bar{\beta}_j\Omega^h$.[41]

I assume that the bilateral trade cost takes the form $\tau_{(j,n)}^h = \left(d_n^h\right)^{\rho_j}(l_n^h)^{-\delta_j^l}(b_n^h)^{-\delta_j^b}\tilde{\epsilon}_{(j,n)}^h$ where bilateral distance, common language and border information are obtained from CEPII's Gravity database. This re-expresses $M_{(j,n)}^h = \rho_j\triangle_n^h - \delta_j^l L_n^h - \delta_j^b B_n^h + \ln\tilde{\epsilon}_{(j,n)}^h$ where $\triangle_n^h \equiv \ln\left(\frac{d_n^h}{\bar{d}^h}\right) - \sum_{n'}\frac{Y^{n'}}{Y^W}\ln\left(\frac{d_n^{n'}}{\bar{d}^{n'}}\right)$ and $\bar{d}^{n'} = \exp\left(\frac{1}{N}\sum_n \ln d_n^{n'}\right)$ and L_n^h and B_n^h are defined in the same way. To separately identify γ_j, I again follow Fajgelbaum and Khandelwal (2016) and set the elasticity of trade cost with respect to distance $\rho^j = \rho = 0.177$.[42]

Recall that $\Omega^h = y^h - \sum_{n'}\left(\frac{Y^{n'}}{Y^W}\right)y^{n'}$ where $y^h = \ln(\frac{\bar{w}^h}{a(p^h)}) + \sum^h$. I proxy the homothetic price aggregator $a(p^h)$ with a Stone index: $a(p^h) = \sum_n S_n^h \ln(p_{nn}(d_n^h)^\rho)$, where p_{nn} are the quality-adjusted prices estimated by Feenstra and Romalis (2014). I obtain estimates of $\bar{w}^h$ and $\sum^h$ from the supply side as reported in the last section.

The estimating equation that I take to the data is the following:

$$\frac{Y_{(j,n)}^h}{Y^h} - \frac{Y_{(j,n)}}{Y^W} = a_n\left(S_j^h - S_j^W\right) - (\gamma_j\rho)\triangle_n^h + \left(\gamma_j\delta_j^l\right)L_n^h + \left(\gamma_j\delta_j^b\right)B_n^h + \tilde{\beta}_{(j,n)}\Omega^h + \epsilon_{(j,n)}^h \tag{80}$$

where $\tilde{\beta}_{(j,n)} = \beta_{(j,n)} - a_n\bar{\beta}_j$. To separately identify $\beta_{(j,n)}$, I need to estimate a_n (in the same equation) and $\bar{\beta}_j = \sum_n \beta_{(j,n)}$ from $S_j^h = \bar{\alpha}_j^h + \bar{\beta}_j y^h = \sum_n \alpha_{(j,n)}^h + \bar{\beta}_j y^h = a_j + \bar{\beta}_j y^h + \varepsilon_j^h$. The left-hand side of the equation is computed from WIOD, using average flows between 2005 and 2007 to smooth out any temporary shocks. In the benchmark, I compute expenditure shares as percentages of total expenditure. As a robustness check, I compute expenditure shares as percentages of final consumption expenditure.

Table 2 reports my estimates of the cross-substitution elasticities between different suppliers of a good within each sector. Note that the sector-level non-homothetic gravity equations add up to a single-sector gravity equation. The sum of my estimates of γ_j across

[41] Please see Appendix 9.10 for the derivation of $K_{(j,n)}^h$.

[42] Alternatively, I can estimate γ_j for each non-service sector separately using tariffs as a trade cost shifter as in Caliando & Parro (2015). Bilateral tariff data at the sector level can be obtained from the UNCTAD-TRAINS.

sectors is 0.24. It is very close to the estimate in Fajgelbaum and Khandelwal (2016). Estimating a translog gravity equation, Novy (2012) reports $\gamma = 0.167$ while Feenstra and Weinstein (2013) reports a median γ of 0.19.

sector	γ_j-total	γ_j-final	sector	γ_j-total	γ_j-final
Agriculture	0.0060	0.0048	Sales, Repair of Motor Vehicles	0.0030	0.0030
Mining	0.0029	0.0008	Wholesale Trade and Comission Trade	0.0115	0.0121
Food, Beverages and Tobacco	0.0086	0.0102	Retail Trade	0.0104	0.0131
Textiles	0.0021	0.0017	Hotels and Restaurants	0.0074	0.0109
Leather and Footwear	0.0004	0.0004	Inland Transport	0.0046	0.0042
Wood Products	0.0013	0.0003	Water Transport	0.0006	0.0001
Printing and Publishing	0.0037	0.0017	Air Transport	0.0013	0.0012
Coke, Refined Petroleum, Nuclear Fule	0.0045	0.0023	Other Auxilliary Transport Activities	0.0025	0.0015
Chemicals and Chemical Products	0.0068	0.0022	Post and Telecommunications	0.0058	0.0051
Rubber and Plastics	0.0026	0.0006	Financial Intermediation	0.0180	0.0102
Other Non-Metallic Minerals	0.0028	0.0007	Real Estate Activities	0.0179	0.0252
Basic Metals and Fabricated Metal	0.0103	0.0021	Renting of M&Eq	0.0158	0.0058
Machinery	0.0047	0.0048	Public Admin and Defense	0.0166	0.0317
Electrical and Optical Equipment	0.0081	0.0048	Education	0.0067	0.0133
Transport Equipment	0.0058	0.0052	Health and Social Work	0.0103	0.0204
Manufacturing, nec	0.0015	0.0019	Other Community and Social Services	0.0101	0.0143
Electricity, Gas and Water Supply	0.0072	0.0042	Private Households with Employed Persons	0.0003	0.0006
Construction	0.0215	0.0364	sum	0.2433	0.2580

Table 2: Cross-substitution between goods

Table 3 reports my estimates of the sectoral income elasticities, $\beta_j = \sum_n \beta_{(j,n)}$. The corresponding elasticities for food, manufacturing and services are -0.022, -0.0051 and 0.0271, respectively. I find that the service sectors have a higher income elasticity as expected.

sector	β_j-total	β_j-final	sector	β_j-total	β_j-final
Agriculture	-0.0128	-0.0117	Sales, Repair of Motor Vehicles	0.0020	0.0022
Mining	-0.0052	-0.0002	Wholesale Trade and Comission Trade	-0.0001	-0.0008
Food, Beverages and Tobacco	-0.0080	-0.0103	Retail Trade	-0.0011	0.0000
Textiles	-0.0034	-0.0024	Hotels and Restaurants	0.0004	0.0016
Leather and Footwear	-0.0005	-0.0004	Inland Transport	-0.0041	-0.0044
Wood Products	-0.0006	0.0002	Water Transport	-0.0008	-0.0012
Printing and Publishing	0.0007	0.0012	Air Transport	0.0003	0.0002
Coke, Refined Petroleum, Nuclear Fule	-0.0017	0.0004	Other Auxilliary Transport Activities	0.0024	0.0011
Chemicals and Chemical Products	-0.0027	-0.0009	Post and Telecommunications	0.0005	0.0002
Rubber and Plastics	-0.0005	-0.0003	Financial Intermediation	0.0117	0.0032
Other Non-Metallic Minerals	-0.0009	0.0000	Real Estate Activities	0.0059	0.00106
Basic Metals and Fabricated Metal	0.0004	0.0004	Renting of M&Eq	0.0131	0.0016
Machinery	-0.0003	-0.0006	Public Admin and Defense	0.0028	0.0051
Electrical and Optical Equipment	-0.0002	-0.0014	Education	0.0012	0.0026
Transport Equipment	-0.0022	-0.0013	Health and Social Work	0.0072	0.0137
Manufacturing, nec	0.0000	0.0002	Other Community and Social Services	0.0005	0.0013
Electricity, Gas and Water Supply	-0.0004	0.0010	Private Households with Employed Persons	0.0002	0.0004
Construction	-0.0038	-0.0111	sum	0.000	0.000

Table 3: Sectoral Betas

Figure 7 plots the sectoral income elasticity computed from total expenditure and final consumption against the exporter's log average income. The correlation coefficient is about 0.4 using either measure. I find a positive relationship which implies that high-income countries specialize in the production of high-income elastic goods, which is consistent with previous findings in Hallak (2006), Khandelwal (2010), Hallak and Schott (2011) and Feenstra and Romalis (2014). The null hypothesis that all income elasticities are zero is rejected.

Figure 8 plots the sectoral income elasticity computed from total expenditure and final consumption against the skill intensity of each sector. I find that skill-intensive sectors produce goods that have a high income elasticity. The correlation coefficient is 0.4 when I use total expenditure to estimate the sectoral income elasticity. This implies that a decline in the relative price of low-income elastic goods from trade liberalization is correlated with a decline in the relative price of goods in unskill-intensive sectors. This implication, along with the other two mentioned in the last section, suggests that trade liberalization increases the nominal wage inequality within a country.

Figure 7: Average Income and Income Elastcity of Production

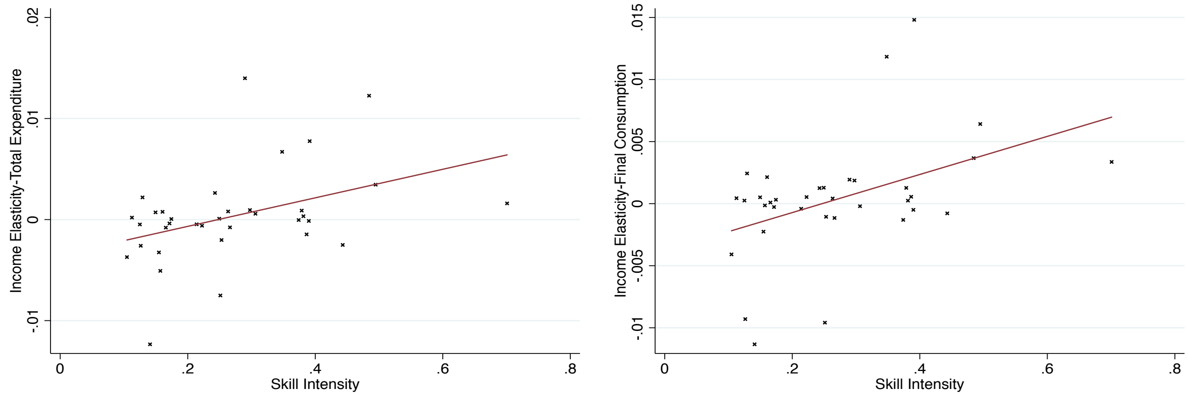

Figure 8: Skill Intensity and Sectoral Income Elastcity

Finally, to estimate $\alpha^h_{(j,n)}$, I assume that it can be decomposed into an exporter effect, a_n, a sector specific effect, a_j and an importer specific taste for that sector, ε^h_j: $\alpha^h_{(j,n)} = a_n(a_j + \varepsilon^h_j)$ as before. I then estimate a_n from the sector-level non-homothetic gravity equation and $a_j + \varepsilon^h_j$ from the sectoral expenditure share: $S^h_j = \bar{\alpha}^h_j + \bar{\beta}_j y^h = \sum_n \alpha^h_{(j,n)} + \bar{\beta}_j y^h = a_j + \bar{\beta}_j y^h + \varepsilon^h_j$.[43]

6 Counterfactuals

Recall that equation (14) can be used to compute the global welfare change of individual z between trade and a counterfactual scenario:

$$\underbrace{u_z^{tr\to cf}}_{\text{total effect}} = \underbrace{\left(\frac{E^h_{cf}}{E^h_{tr}}\right)}_{\text{agg. exp. effect}} \underbrace{\left(\frac{w_z^{tr}}{\tilde{w}^h_{tr}}\right)^{-\ln(b^h_{cf}/b^h_{tr})}}_{\text{ind. exp. effect}} \underbrace{\left(\frac{w_z^{cf}}{w_z^{tr}}\right)}_{\text{income effect}}$$

$\frac{E^h_{cf}}{E^h_{tr}} = \prod_{(j,n)} \left(\frac{p^{h,tr}_{(j,n)}}{p^{h,cf}_{(j,n)}}\right)^{S^h_{(j,n)}}$ is the aggregate expenditure effect, and it measures the reduction in the price index for a country's representative consumer. $\left(\frac{w^{tr}_z}{\tilde{w}^h_{tr}}\right)^{-\ln(b^h_{cf}/b^h_{tr})}$ is the individual expenditure effect, where $-\ln\left(\frac{b^h_{cf}}{b^h_{tr}}\right) = -\sum_j \sum_n \beta_{(j,n)} \ln\left(\frac{p^{h,cf}_{(j,n)}}{p^{h,tr}_{(j,n)}}\right)$. For individual z who is richer than the representative consumer, a decrease in the relative price of low-income

[43] Alternatively, note that the aggregate expenditure share in equation (11) is a non-linear function in $\alpha^h_{(j,n)}$ and, $\{p^h_{(j,h)}\}_{j\in J}$ $\forall h$, the output prices in the general equilibrium, given the estimates of γ_j and $\beta_{(j,n)}$. I use $S^h_{(j,n)}$ as an initial guess for $\alpha^h_{(j,n)}$ and solve for the prices. Given these prices, I solve for an updated value of $\alpha^h_{(j,n)}$, which is used in the next iteration, and the procedure continues until convergence.

elastic goods makes her better off. $\frac{w_z^{cf}}{w_z^{tr}}$ is the income effect, and its change depends on the sector that individual z works in. An increase in a sector's output price raises the relative nominal wage of the labor groups that disproportionately work in that sector in the initial trade equilibrium.

6.1 Five Percent Reduction in Trade Costs

I first consider a simultaneous 5% reduction in all bilateral trade costs, starting from the baseline parametrization.[44] Since I am interested in the impact of trade liberalization on different groups of people, in particular, the poor versus the rich, I focus on the difference in welfare change between the 10th percentile and the 90th percentile of the initial nominal wage distribution within each country that comes from each of the components in equation (14). Since the aggregate expenditure effect is the same for every individual within a country, it is differenced out. I define the following terms: **diff. exp. effect** = ind. exp. effect$_{z=10th}-$ ind. exp. effect$_{z=90th}$; **diff. inc. effect** = income effect$_{z=10th}-$ income effect$_{z=90th}$; **diff. tot. effect** = total effect$_{z=10th}-$ total effect$_{z=90th}$.

6.1.1 Income Channel

I first study the distributional effects of trade liberalization through the income channel. The second column of Table 4 reports the lower and upper bounds of **diff. exp. effect**, **diff. inc. effect**, **diff. tot. effect** across the 40 countries when only the income channel is active.[45] I shut down the expenditure channel by imposing that $\beta_{(j,n)} = 0 \forall j \in J$, $n \in N$. This brings me back to a translog demand system, which is homothetic. Under these restrictions, the consumer price index for every individual within a country changes by the same amount, i.e. **diff. exp. effect**=0.[46]

[44]Please see Appendix 9.11 for a discussion about how to implement the counterfactual where each country moves back to autarky.

[45]I calibrate the model by assigning values to its parameters, that is, the standard error of the estimate is assume to be 0 for all parameters. As a result, the standard errors of the bounds are also 0.

[46]To conduct counterfactuals based on the restricted model, I re-calibrate parameters with the additional assumptions. More specifically, in the case where $\beta_{(j,n)} = 0\ \forall j \in J$, $n \in N$, I re-estimate $\{\gamma_j\}_{j\in J}$ using equation (36). In the case where $T(\lambda, j) = 1\ \forall \lambda \in \Lambda$, $j \in J$, I re-calculate $A^h(\lambda)$ using equation (30).

Active channel(s)	Income	Expenditure	Both
diff. exp. effect	0	$[0.43, 0.88]$	$[0.76, 1.36]$
diff. inc. effect	$[-0.01, 0.04]$	0	$[-0.72, -0.04]$
diff. tot. effect	$[-0.01, 0.04]$	$[0.43, 0.88]$	$[0.24, 1.29]$

Table 4: Distributional Effects through Income Channel

I find that in Estonia, the 10th percentile suffers a decrease in the nominal wage relative to the 90th percentile of 0.01 percentage points. On the other hand, in Portugal, the 10th percentile enjoys an increase in the relative nominal wage of 0.04 percentage points. The change in the relative nominal wage in the rest of the countries lies in between.

Panel A of Figure 9 plots **diff. inc. effect** against the log average income for each country based on a weighted least squares regression with weights equal to the output share of a country in the world economy. The correlation coefficient is -0.18. Panel B plots a country's skill abundance against its log average income. The correlation coefficient is 0.77.[47] I find that the income channel benefits the poor more than the rich in low-income countries that are skill-scarce. These countries have a comparative advantage in unskill-intensive sectors and a reduction in trade costs increases the relative nominal wage of the poor because they are less skilled and more likely to work in unskill-intensive sectors. On the other hand, the income channel benefits the rich more than the poor in high-income countries that are skill-abundant. These countries have a comparative advantage in skill-intensive sectors and a reduction in trade costs increases the relative nominal wage of the rich because they are more skilled and more likely to work in skill-intensive sectors.

[47] I measure a country's skill abundance, $H_n/(H_n + L_n)$, as the share of workers with a completed tertiary degree (i.e. university graduates and post-graduates).

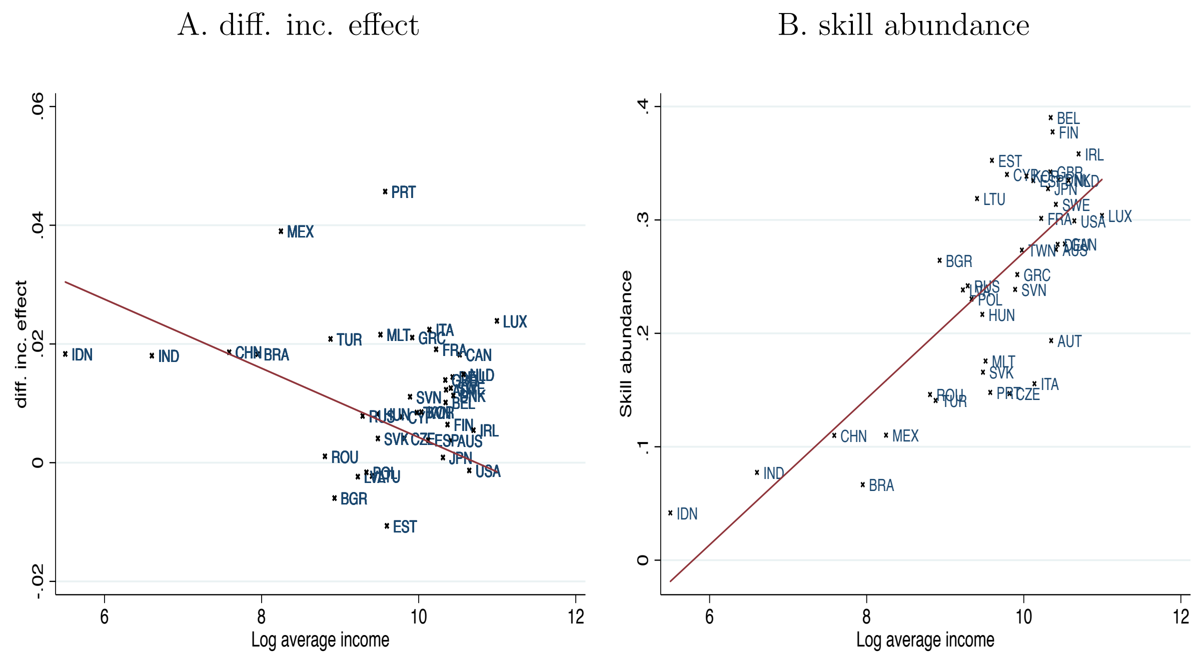

Figure 9: Distributional Effects through Income Channel

6.1.2 Expenditure Channel

I next study the distributional effects of trade liberalization through the expenditure channel. The third column of Table 4 reports the lower and upper bounds of **diff. exp. effect**, **diff. inc. effect**, **diff. tot. effect** across the 40 countries when only the expenditure channel is active. I shut down the income channel by imposing that $T(\lambda, j) = 1\,\forall \lambda \in \Lambda,\, j \in J$, that is, there is no comparative advantage of different labor types across sectors. Under these restrictions, the nominal wage of every individual within a country changes by the same amount, i.e. **diff. inc. effect**=0.

Active channel(s)	Income	Expenditure	Both
diff. exp. effect	0	$[0.43, 0.88]$	$[0.76, 1.36]$
diff. inc. effect	$[-0.01, 0.04]$	0	$[-0.72, -0.04]$
diff. tot. effect	$[-0.01, 0.04]$	$[0.43, 0.88]$	$[0.24, 1.29]$

Table 4: Distributional Effects through Expenditure Channel

I find that the expenditure channel benefits the poor more than the rich in every country. More specifically, in Indonesia, the 10th percentile enjoys a reduction in consumer price index that is 0.43 percentage points bigger than the 90th percentile. On the other hand, in Taiwan, the 10th percentile enjoys a reduction in consumer price index that is

0.88 percentage points bigger than the 90th percentile. The poor's relative benefit from the expenditure channel in the rest of the countries lies in between.

Why does the expenditure channel imply a pro-poor bias in every country? The most direct effect of a reduction in trade costs is to decrease $a\left(\mathbf{p}^h\right)$, the homothetic price aggregator, which increases the inequality-adjusted real wage, $\ln\left(\frac{\tilde{w}^h}{a\left(\mathbf{p}^h\right)}\right)$, in every country h, and therefore decreases the expenditure shares on goods with $\beta_{(j,n)} < 0$. This is an inward shift in the demand for low-income elastic goods which decreases their relative price. Since low-income consumers spend more on these goods, they benefit more from the expenditure channel.

Figure 10 plots the percentage change in the price of each of the $J * N = 1400$ goods against its income elasticity, $\beta_{(j,n)}$. Panel A uses the income elasticity computed from total expenditure while Panel B restricts to final consumption. The correlation is strongly positive regardless of which estimate of $\beta_{(j,n)}$ I use, that is, there is a decrease in the relative price of low-income elastic goods following trade liberalization.

$$\left(p^{cf}_{(j,n)} - p^{tr}_{(j,n)}\right) \Big/ p^{tr}_{(j,n)}$$

Panel A Panel B

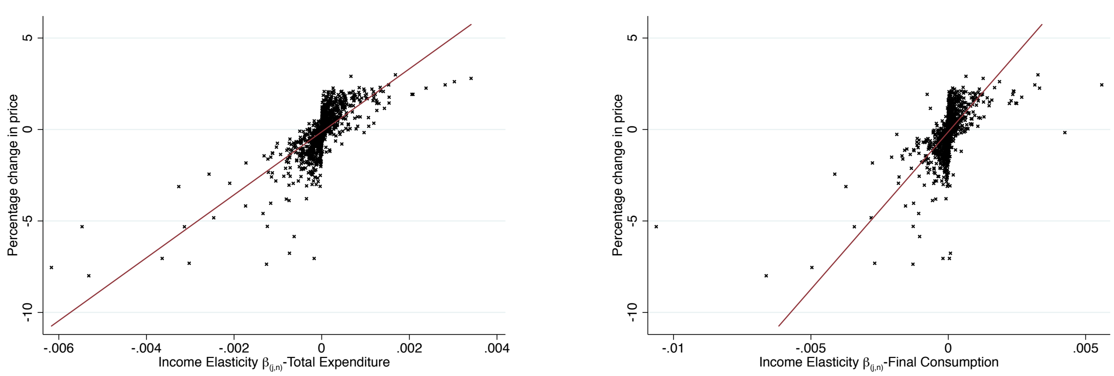

Figure 10: Percentage Change in Price

Across countries, I find that expenditure channel benefits the poor relative to the rich even more in high-income countries that import low-income elastic goods. Panel A of Figure 11 plots **diff. exp. effect** against the log average income for each country based on a weighted least squares regression with weights equal to the output share of a country in the world economy. The correlation coefficient is 0.37. Panel B plots the income elasticity of a country's imports relative to its production against its log average income. The correlation coefficient is -0.30.[48] Because high-income countries import low-income elastic goods, the decrease in the relative price of low-income elastic goods is magnified by the lower trade costs, which implies a bigger relative benefit from the expenditure channel for the poor.

A. diff. exp. effect B. income elasticity of imports

Figure 11: Distributional Effects through Expenditure Channel

6.1.3 Both Channels

Finally, I study the distributional effects of trade liberalization through both channels. The average gain from a simultaneous 5% reduction in all bilateral trade costs across the countries is 1.2%. As one moves up the income distribution, gains decline. More specifically, moving to the next decile reduces gains by about 0.1 percentage points. The third column of Table 4 reports the lower and upper bounds of **diff. exp. effect**, **diff. inc. effect**, **diff. tot. effect** across the 40 countries when both the expenditure channel and the income channel are active. Since non-homothetic preferences allow people with different incomes to consume different bundles of goods, price changes resulting from trade liberalization can have a differential impact on an individual's consumer price index. I find a pro-poor bias from the expenditure channel in every country, i.e., **diff. exp. effect**>0. On average, the 10th percentile sees her consumer price index decrease by 1 percentage point more than the 90th percentile. In addition, since different labor groups

[48] I calculate the income elasticity of a country's imports relative to its production as $\bar{\beta}^h_{imp} - \bar{\beta}^h_{prod} = \sum_j \sum_{n \neq h} \beta_{(j,n)} S^h_{(j,n)} - \sum_j \beta_{(j,h)}$.

sort into different sectors based on comparative advantage, price changes resulting from trade liberalization can have a differential impact on an individual's nominal wage. I find a pro-rich bias from the income channel in every country, i.e., **diff. inc. effect**<0. On average, the 10th percentile sees her nominal wage go down by 0.24 percentage points relative to the 90th percentile.[49] Since the expenditure effect dominates the income effect in magnitude, trade liberalization benefits the poor more than the rich in every country, i.e., **diff. tot. effect**>0. I find that in Luxembourg, the 10th percentile enjoys an increase in the real wage relative to the 90th percentile of 0.24 percentage points. On the other hand, in Taiwan, the 10th percentile enjoys an increase in the relative real wage of 1.29 percentage points. The poor's relative benefit from both channels in terms of real wages in the rest of the countries lies in between. On average, the difference between the 10th and the 90th percentiles is about 0.8 percentage points.

Active channel(s)	Income	Expenditure	Both
diff. exp. effect	0	$[0.43, 0.88]$	$[0.76, 1.36]$
diff. inc. effect	$[-0.01, 0.04]$	0	$[-0.72, -0.04]$
diff. tot. effect	$[-0.01, 0.04]$	$[0.43, 0.88]$	$[0.24, 1.29]$

Table 4: Distributional Effects through Both Channels

More interestingly, I find that when both channels are active, the poor enjoy an even bigger relative reduction in consumer price indices in every country compared to the case where only the expenditure channel operates, that is, the range of **diff. exp. effect** across the 40 countries changes from $[0.43, 0.88]$ to $[0.76, 1.36]$. In addition, the poor now suffer a relative decrease in nominal wages in every country. Note that the range of **diff. inc. effect** across the 40 countries changes from $[-0.01, 0.04]$ to $[-0.72, -0.04]$. That is, the interaction of the two channels quantitatively changes the prediction of the differential impact of trade liberalization on the poor versus the rich through the expenditure channel and qualitatively through the income channel.[50]

[49] I find a rise in nominal wage inequality in every country, which is qualitatively consistent with a wide range of empirical evidence. The impact of a 5% reduction in trade costs is a small change in relative wage, but a significant change in trade costs could have a big effect. It is also straightforward in the context of my model to introduce skill-biased technological change at the aggregate level. In this paper, I focus on the impact of a change in trade costs holding technology fixed.

[50] I assume that all final good sectors are tradable and consider a decrease in trade costs in all sectors to understand how the model works. Empirical findings suggest that non-tradable sectors typically have a higher income elasticity. Consequently, allowing for non-tradability is expected to decrease the relative nominal wage of the poor and the relative price index for the poor. If the expenditure channel dominates the income channel in terms of magnitude as before, then the poor still benefit more from trade liberalization than the rich in every country.

To see the comparison visually, Figure 12 plots (using blue x) **diff. inc. effect** when only the income channel is active against **diff. exp. effect** when only the expenditure channel is active, and then plots (using red diamond) **diff. inc. effect** against **diff. exp. effect** when both channels are active and interact. The interaction changes the estimates of both effects significantly. More specifically, each country moves to the right which implies that the poor's relative benefit from the expenditure channel is bigger. Also, each country moves downward and **diff. inc. effect**<0 for all of them, which implies that the rich benefit relative to the poor from the income channel in every country.

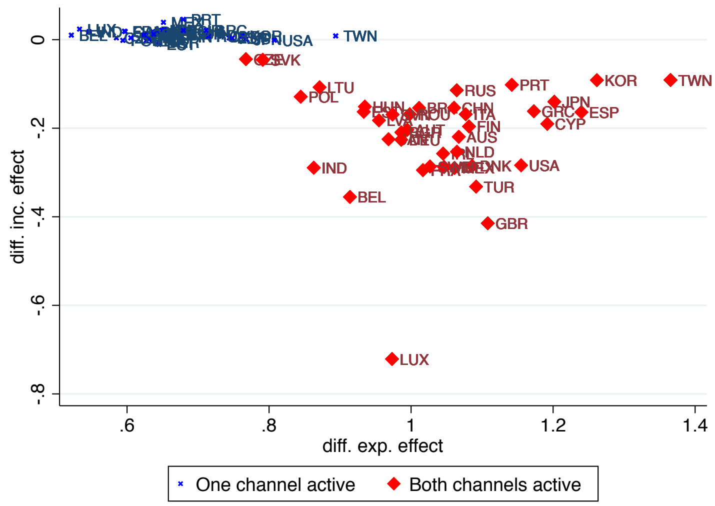

Figure 12: Interaction of Channels

Why does the expenditure channel imply a bigger pro-poor bias and the income channel imply a pro-rich bias in every country? When both channels are active, lower trade costs reduce the relative demand for and the relative price of low-income elastic goods as discussed before. However, since the poor disproportionately produce unskill-intensive goods which are low-income elastic, their relative nominal wage goes down in every country. This implies that the income channel benefits the rich everywhere. This effect is absent when only the income channel is active because the income elasticity of every good is 0. On the other hand, as the nominal wage inequality goes up, the relative demand for and the relative price of low-income elastic goods fall even further, reducing the relative price index for the poor in every country. This implies that the expenditure channel benefits the poor even more compared to the case where only the expenditure channel is active. This effect is absent in that case because nominal wage inequality is constant.

How does the poor's relative benefit from the combined effect of trade liberalization vary across countries? Figure 13 plots **diff. tot. effect** against the log average income for each country based on a weighted least squares regression with weights equal to the output share of a country in the world economy. The correlation coefficient is 0.19. Since the expenditure channel benefits more the poor individuals in rich countries and the rich individuals in poor countries, while the income channel benefits more the rich individuals in rich countries and the poor individuals in poor countries, allowing both channels to operate no longer makes income per capita a good predictor of the pro-poor bias of trade liberalization.[51]

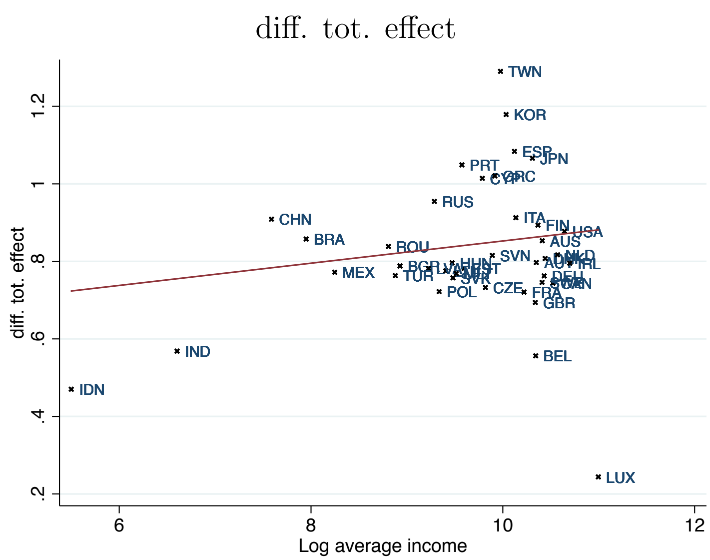

Figure 13: Distributional Effects through Both Channels

6.1.4 Bias from Considering Two Channels Separately

Table 5 reports the bias from considering the two channels separately for each country. In the second column, I add up **diff. inc. effect** when only the income channel is active and **diff. exp. effect** when only the expenditure channel is active, and then compare to **diff. tot. effect** when both channels are active as reported in the third column. I find that estimating the two effects separately and adding them up generates a significant downward bias in the prediction for the poor's relative benefit from trade liberalization.

In particular, this underestimation is stronger in a country like Japan, which produces

[51]Since country characteristics are all correlated and pull in different directions, none in the data that I target has significant explanatory power for the variation in the model's predicted pro-poor bias of trade liberalization across countries.

Country	Separate	Combined	Country	Separate	Combined
AUS	0.75	0.85	IRL	0.66	0.80
AUT	0.66	0.80	ITA	0.68	0.91
BEL	0.55	0.56	JPN	0.76	1.06
BGR	0.64	0.79	KOR	0.76	1.18
BRA	0.66	0.86	LTU	0.63	0.78
CAN	0.63	0.74	LUX	0.60	0.24
CHN	0.71	0.90	LVA	0.63	0.78
CYP	0.74	1.01	MEX	0.70	0.77
CZE	0.59	0.73	MLT	0.68	0.76
DEU	0.65	0.76	NLD	0.67	0.82
DNK	0.68	0.81	POL	0.60	0.72
ESP	0.75	1.08	PRT	0.73	1.05
EST	0.64	0.78	ROU	0.63	0.84
FIN	0.68	0.89	RUS	0.72	0.95
FRA	0.63	0.72	SVK	0.61	0.76
GBR	0.71	0.69	SVN	0.64	0.82
GRC	0.73	1.02	SWE	0.65	0.74
HUN	0.63	0.80	TUR	0.70	0.76
IDN	0.46	0.47	TWN	0.89	1.29
IND	0.57	0.57	USA	0.80	0.87

Table 5: Bias from Considering Two Channels Separately

high-income elastic goods, compared to a country like Mexico, which produces low-income elastic goods. This pattern generalizes to the entire sample of 40 countries. Figure 14 plots the difference in the poor's relative benefit from trade liberalization between estimating the two effectes jointly and separately against the income elasticity of the country's production, $\bar{\beta}^h_{prod} = \sum_j \beta_{(j,h)}$. Panel A uses the income elasticity computed from total expenditure while Panel B is restricted to final consumption. The correlation is strongly positive regardless of which estimate of $\bar{\beta}^h_{prod}$ I use, that is, the interaction of the two channels benefits more the countries that produce high-income elastic goods.[52]

[52]Luxembourg is an outlier. It is one of the smallest sovereign states in Europe and has the world's highest GDP per capita. It also has the highest trade share in my sample of countries. I show in Appendix 9.12 that this correlation remains positive and significant after excluding Luxembourg.

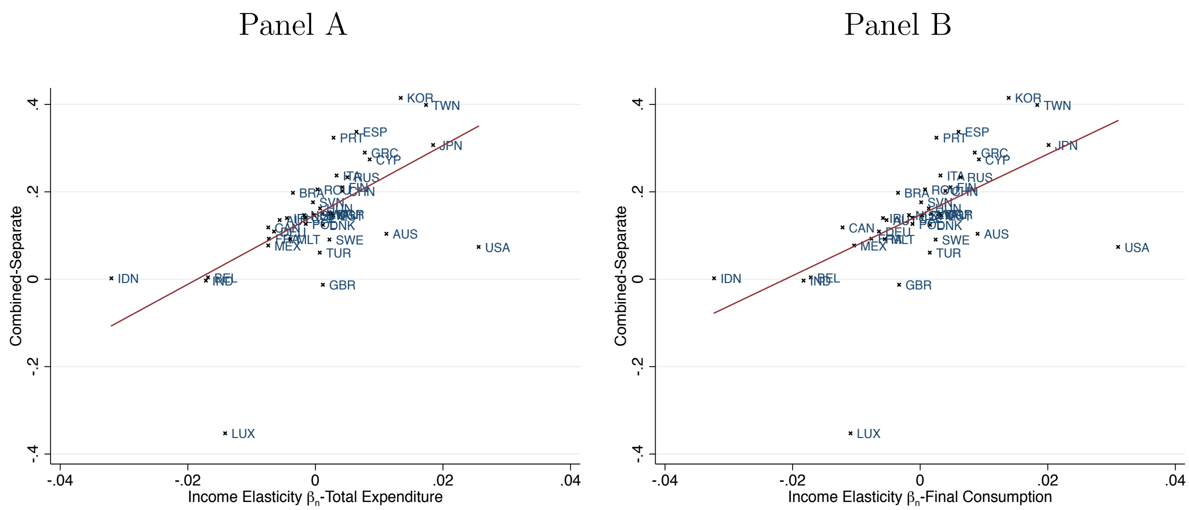

Figure 14: Underprediction of Pro-poor Bias of Trade Liberalization

Intuitively, the interaction reallocates workers away from unskill-intensive sectors that produce low-income elastic goods in every country because it decreases the relative price of low-income elastic goods. However, this is already the case in the countries that specialize in the production of high-income elastic goods without the interaction. Therefore, the interaction induces a smaller increase in worker reallocation away from unskill-intensive sectors in these countries, which implies a bigger benefit for the poor who work in these sectors.

6.2 Rising Chinese Import Competition

Autor, Dorn and Hanson (2013) analyze the effect of rising Chinese import competition between 1990 and 2007 on U.S. local labor markets, and they find that it causes higher unemployment, lower labor force participation, and reduced wages in local labor markets that serve import-competing manufacturing industries.[53][54] They instrument for the growth in U.S. imports from China using Chinese import growth in other high-income markets to isolate the foreign-supply-driven component of the changes, i.e., China's productivity growth and falling trade costs. In particular, for their base specifications, they focus on a single channel through which trade with China affects a region: greater import competition in the U.S. market. This ignores the effects of greater U.S. exports to China

[53] Wage changes in Autor, Dorn and Hanson (2013) are in nominal and not real terms.

[54] It would be interesting and important to introduce unemployment or search into my framework. There would then be consequences about adjustment to trade shocks in the short- and medium-run. I leave it for future work. Please refer to Joan Monras' and Matthieu Bellon's work on these topics.

or greater import competition in the foreign markets that U.S. regions serve. Their main measure of local labor market exposure to import competition is the change in Chinese import exposure per worker in a region, where imports are apportioned to the region according to its share of national industry employment. They also control for the start-of-period manufacturing share within commuting zones so as to focus on variation in exposure to Chinese imports stemming from differences in industry mix within local manufacturing sectors.

Instead of using the variation across local labor markets, I analyze the aggregate effect of a \$1K increase in U.S. manufacturing imports from China per worker.[55] At initial equilibrium, average per capita spending by the U.S. on Chinese manufacturing goods is $\sum_{j\in M} S^{us}_{(j,chn)}\bar{w}^{us} = 0.0187*22.4128 = 0.42$.[56] To increase it by \$1K is equivalent to an increase in the total expenditure share on these goods of 4.46%.[57] I shut down the effects of greater U.S. exports to China or greater import competition in the foreign markets that the U.S. serves by holding the production prices, $p_{(j,h)}\,\forall j \in J$, $\forall h \neq US$, and trade costs, $\tau^n_{(j,h)}\,\forall j \in J$, $\forall h \neq CHN$, $\forall n \neq US$, unchanged. To compute the reduction in trade costs in the manufacturing sectors that would lead to this increase in Chinese imports, I apply the expenditure share equation (32), and it follows that $\sum_{j\in M} \triangle S^{us}_{(j,chn)} = \sum_{j\in M} \frac{(1-N)\gamma_j}{N} \ln(\partial\tau)$ where $\tau^{us,cf}_{(j,chn)} = \tau^{us,tr}_{(j,chn)} \cdot \partial\tau$ if $j \in M$.[58] Plugging in the estimates of γ_j, I have $\ln(\partial\tau) = -0.8$. Applying equation (32) again, I calculate the impact of this reduction in trade costs on U.S. expenditure shares on domestic goods, $\triangle S^{us}_{(j,us)} = \frac{\gamma_j}{N}\ln(\partial\tau)$. I then solve for U.S. production prices again, $p^{us}_{(j,us)}\,\forall j \in J$, such that the U.S. market clearing conditions (equation (26)) are still satisfied, taking into account the change in domestic demand.

I find that production prices, $p^{us}_{(j,us)}$, go down in all $j \in J$ in the U.S. as a result of rising Chinese import competition. They decrease in the manufacturing sectors because of the

[55] 1 unit in my framework is approximately \$1000.

[56] Sectors "Agriculture" and "Food, Beverages and Tobacco" are the food sectors; "Mining" and from "Textiles" to "Manufacturing, nec" in the first column in Table 2 and 3 are the manufacturing sectors. The remaining sectors are the service sectors.

[57] $\triangle \sum_{j\in M} S^{us}_{(j,chn)} \cdot \bar{w}^{u} = 1 \rightarrow \triangle \sum_{j\in M} S^{us}_{(j,chn)} = 1/\bar{w}^{us} = 0.0446$. Note that this increase in spending on Chinese goods that Autor, Dorn and Hanson (2013) consider is due to supply and trade-cost-driven changes in China's export performance, not changes in US import demand as a result of higher income.

[58] Note that I attribute this increase in Chinese imports entirely to the reduction in trade costs for simplicity. Suppose it is due to China's improved productivity instead, then its production prices would decrease. Both of these forces have the same effect on US consumer prices, each of which is the product of the production price and the trade cost. Note also that the change in these trade costs also affects y^{us} through its impact on $a(p^{us})$. I ignore it since this effect is negligibly small and does not change the result of the analysis.

lower demand for the domestically produced goods, and in the non-manufacturing sectors because workers choose to leave manufacturing and work in other sectors in response to lower output prices and wages in manufacturing. This increases the labor supply in the non-manufacturing sectors, putting downward pressure on the output prices in these sectors. The aggregate expenditure effect, $\frac{E^{us}_{cf}}{E^{us}_{tr}}$, is 0.85%, that is, the reduction in the cost of Chinese manufacturing imports decreases the consumer price index for a U.S. representative consumer by 0.85%.[59] The individual expenditure effect, $\left(\frac{w_z}{\tilde{w}^{us}}\right)^{-\ln(b^{us}_{cf}/b^{us}_{tr})}$, implies a pro-poor bias of 0.45 percentage points, with an individual whose wage is at the 10th percentile of the initial distribution sees a further 0.35 percentage points reduction in her consumer price index compared to the representative consumer and an individual whose income is at the 90th percentile sees her consumer price index decrease by 0.1 percentage points less than the representative consumer. This result comes from the fact that Chinese manufacturing goods are low-income elastic and, consequently, their lower prices benefit more the poor individuals who spend relatively more on these goods.[60] The income effect, $\left(\frac{w^{cf}_z}{w_z}\right)$, implies a pro-rich bias of 0.02 percentage points, while a poor and unskilled worker sees her nominal wage go down by 0.13% and a rich and skilled worker sees her nominal wage go down by 0.11%. The reason that the former sees a bigger decline in her nominal wage is because she is more likely to work in manufacturing sectors that are in direct competition with cheaper Chinese imports. The more pronounced decrease in the output prices in these sectors leads to the bigger decrease in her nominal wage. Combining all three effects, a poor individual gains 0.43 percentage points more compared to a rich one in terms of real wage as a result of the rising Chinese import competition. That is, the pro-rich bias of the income effect is more than offset by the pro-poor bias of the expenditure effect which again underlines the importance of taking both channels into account in assessing the distributional effects of trade liberalization.

7 Conclusion

What is the impact of trade liberalization on the distribution of real wages in a large cross-section of countries? The vast majority of the literature focuses on the effect of trade on the distribution of nominal wages. A small number of studies consider its differential

[59] 0.75% of this decrease stems from the lower consumer prices of Chinese imports in the U.S. and the remaining 0.1% comes from the lower production prices of U.S. goods.

[60] 11 out of China's 14 manufacturing sectors have $\beta_{(j,chn)} < 0$.

impact on consumer price indices. To my knowledge, there are only three case studies that have combined both channels to examine how real wages of different groups of people are affected in individual countries, Argentina, Mexico and India.

I build a model combining demand heterogeneity across consumers with productivity heterogeneity across workers to quantify the distributional effects of trade liberalization for a wide range of countries taking both channels into account. By looking at a large set of countries, I am able to identify general patterns across countries with different characteristics. I am also able to conduct model-based counterfactuals of different trade shocks, which are important for policymakers. I use sector-level trade and production data to estimate the parameters of the model. I find that as a result of a five percent reduction in all bilateral trade costs, the bigger decline in the poor's consumer price indices more than compensates for their lower relative nominal wage. More specifically, in the average country, real wage of the bottom 10th percentile increases by 0.8 percentage points more than the top 10th percentile. I also find that there is an important interaction between the two channels, and therefore, estimating the two effects separately and adding them up leads to a significant bias. These results highlight the importance of combining both channels in order to measure the distributional effects of trade accurately.

My findings have important policy implications for the distribution of winners and losers from trade reforms. There has been increasing public resistance to freer trade that originates from the belief that the most vulnerable group, i.e., the poor and unskilled, will be hurt the most. This paper demonstrates that such a belief is misguided.

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9 Appendix

9.1 Welfare Change as Equivalent Variation

Consider the set of changes $\left\{\widehat{p^h_{(j,n)}}\right\}_{(j,n)\in\mathcal{J}\times\mathcal{N}}$ and $\{\widehat{w_z}\}_{z\in\mathcal{Z}_h}$. The resulting change in the indirect utility is:

$$\widehat{v_z} = \sum_j \sum_n \frac{\partial \ln v\left(w_z, \mathbf{p}^h\right)}{\partial \ln p^h_{(j,n)}} \widehat{p^h_{(j,n)}} + \frac{\partial \ln v\left(w_z, \mathbf{p}^h\right)}{\partial \ln w_z} \widehat{w_z}$$

The equivalent variation $\widehat{u_z}$ is the proportional change in income at the original prices to induce the same proportional change in indirect utility:

$$\widehat{v_z} = \frac{\partial \ln v\left(w_z, \mathbf{p}^h\right)}{\partial \ln w_z} \widehat{u_z}$$

They imply, with the help of Roy's identity,

$$\widehat{u_z} = \widehat{w_z} + \sum_j \sum_n \left(\frac{\partial \ln v\left(w_z, \mathbf{p}^h\right)}{\partial \ln p^h_{(j,n)}} \Big/ \frac{\partial \ln v\left(w_z, \mathbf{p}^h\right)}{\partial \ln w_z} \right) \widehat{p^h_{(j,n)}} = \widehat{w_z} + \sum_j \sum_n -s^z_{(j,n)} \widehat{p^h_{(j,n)}}$$

9.2 Welfare Change

Integrate the aggregate expenditure effect $\widehat{E^h} = \sum_j \sum_n S^h_{(j,n)} \left(-\widehat{p^h_{(j,n)}}\right)$,

$$\int \partial \ln E^h = \sum_j \sum_n S^h_{(j,n)} \left(-\int \partial \ln p^h_{(j,n)}\right) \;\; \rightarrow \;\; \ln E^h = -\sum_j \sum_n \ln\left[(p^h_{(j,n)})^{S^h_{(j,n)}} \right]$$

$$E^h = \exp\left(-\sum_j \sum_n \ln\left[(p^h_{(j,n)})^{S^h_{(j,n)}} \right]\right) \;\; = \;\; \prod_{(j,n)} \exp(-\ln[(p^h_{(j,n)})^{S^h_{(j,n)}}]) = \prod_{(j,n)} (p^h_{(j,n)})^{-S^h_{(j,n)}}$$

As a result,

$$\frac{E^h_{cf}}{E^h_{tr}} \;\; = \;\; \prod_{(j,n)} \left(\frac{p^{h,tr}_{(j,n)}}{p^{h,cf}_{(j,n)}}\right)^{S^h_{(j,n)}}$$

Integrate the individual expenditure effect $\widehat{b^h} = \sum_j \sum_n \beta_{(j,n)} \widehat{p^h_{(j,n)}}$, $b^h = \prod_{(j,n)} (p^h_{(j,n)})^{\beta_{(j,n)}}$

$$\frac{b^h_{cf}}{b^h_{tr}} = \prod_{(j,n)} (\frac{p^{h,cf}_{(j,n)}}{p^{h,tr}_{(j,n)}})^{\beta_{(j,n)}} \quad \rightarrow \quad -\ln(\frac{b^h_{cf}}{b^h_{tr}}) = -\sum_j \sum_n \beta_{(j,n)} \ln(\frac{p^{h,cf}_{(j,n)}}{p^{h,tr}_{(j,n)}})$$

9.3 Specialization in Production

I construct an index of a country $n's$ relative supply of goods in skill-intensive sectors as the following: $\frac{\sum_{j=1}^{J} \alpha_j supply(j,n)}{\sum_{j=1}^{J} supply(j,n)}$ where α_j denotes the skill intensity of a sector. As expected, skill-abundant countries produce relatively more in skill-intensive sectors at equilibrium. In addition, I construct an index of a country $n's$ relative price increase in skill-intensive sectors as the following: $\sum_j \left[(p^{cf}_{(j,n)} - p^{tr}_{(j,n)})/p^{tr}_{(j,n)} \right] \alpha_j$. As expected, I find that skill-abundant countries see a bigger increase in the relative price of skill-intensive goods after trade liberalization.

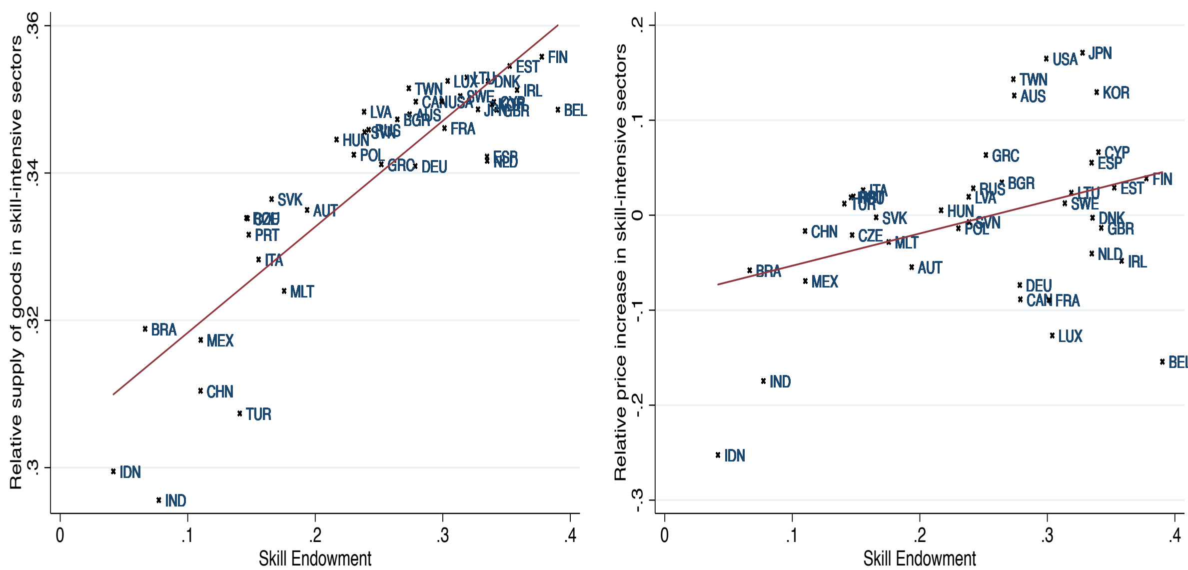


Figure 15: Specialization in Production and Price Changes

9.4 Total Supply

Output produced by a worker of labor type λ who works in sector j in country h is:

$$A^h(\lambda)T(\lambda, j)\boldsymbol{E}\left(\epsilon_z | z \in \mathcal{Z}^h(\lambda), w_z(j) \geq w_z(j')\right) \forall j' \in J$$

$$= A^h(\lambda)T(\lambda,j)\frac{1}{\pi^h(\lambda,j)}\int_0^\infty \epsilon(z,j)Pr\left(\epsilon(z,j) \geq max_{j'\neq j}\epsilon(z,j')\frac{x^h(\lambda,j')}{x^h(\lambda,j)}\right)dG(\epsilon)$$

$$Pr\left(\epsilon(z,j) \geq max_{j'\neq j}\epsilon(z,j')\frac{x^h(\lambda,j')}{x^h(\lambda,j)}\right) = \quad \Pi_{j'\neq j}Pr\left(\epsilon(z,j') \leq \epsilon(z,j)\tfrac{x^h(\lambda,j)}{x^h(\lambda,j')}\right)$$

$$= \Pi_{j'\neq j}\exp\Bigg(-\epsilon(z,j)^{-\theta(\lambda)}x^h(\lambda,j)^{-\theta(\lambda)}x^h(\lambda,j')^{\theta(\lambda)}\Bigg)$$

$$= \exp\left(-\epsilon(z,j)^{-\theta(\lambda)}x^h(\lambda,j)^{-\theta(\lambda)}\sum_{j'\neq j}x^h(\lambda,j')^{\theta(\lambda)}\right)$$

$$G\Bigg(\epsilon(z,j),\lambda\Bigg) = \exp\Bigg(-\epsilon(z,j)^{-\theta(\lambda)}\Bigg)$$

$$\partial G\Bigg(\epsilon(z,j),\lambda\Bigg) = \exp\Bigg(-\epsilon(z,j)^{-\theta(\lambda)}\Bigg)\theta(\lambda)\epsilon(z,j)^{-\theta(\lambda)-1}\partial\epsilon$$

$$A^h(\lambda)T(\lambda,j)\boldsymbol{E}\Bigg(\epsilon_z|z\in\mathcal{Z}^h(\lambda), w_z(j)\geq w_z(j')\,\forall j'\in J\Bigg)$$

$$= A^h(\lambda)T(\lambda,j)\frac{1}{\pi^h(\lambda,j)}\int_0^\infty \epsilon(z,j)\exp\left(-\epsilon(z,j)^{-\theta(\lambda)}x^h(\lambda,j)^{-\theta(\lambda)}\sum_{j'\neq j}x^h(\lambda,j')^{\theta(\lambda)}\right)$$

$$\exp\Bigg(-\epsilon(z,j)^{-\theta(\lambda)}x^h(\lambda,j)^{-\theta(\lambda)}x^h(\lambda,j)^{\theta(\lambda)}\Bigg)\theta(\lambda)\epsilon(z,j)^{-\theta(\lambda)-1}\partial\epsilon$$

$$= A^h(\lambda)T(\lambda,j)\frac{1}{\pi^h(\lambda,j)}\int_0^\infty \exp\left(-\epsilon(z,j)^{-\theta(\lambda)}x^h(\lambda,j)^{-\theta(\lambda)}\sum_{j'\in J}x^h(\lambda,j')^{\theta(\lambda)}\right)\theta(\lambda)\epsilon(z,j)^{-\theta(\lambda)}\partial\epsilon$$

$$= A^h(\lambda)T(\lambda,j)\frac{1}{\pi^h(\lambda,j)}\int_0^\infty \exp\left(-\epsilon(z,j)^{-\theta(\lambda)}\frac{1}{\pi^h(\lambda,j)}\right)\theta(\lambda)\epsilon(z,j)^{-\theta(\lambda)}\partial\epsilon$$

$$Let\, r = \epsilon(z,j)^{-\theta(\lambda)}\frac{1}{\pi^h(\lambda,j)}. \quad Then\, \partial r = \tfrac{1}{\pi^h(\lambda,j)}\left(-\theta(\lambda)\right)\epsilon(z,j)^{-\theta(\lambda)-1}\partial\epsilon.$$

$$Recall\, that\, \Gamma(t) \;=\; \int_0^\infty r^{t-1}e^{-r}\partial r,$$

$$\Gamma(1-\frac{1}{\theta(\lambda)}) = \int_0^\infty \exp\left(-\epsilon(z,j)^{-\theta(\lambda)}\frac{1}{\pi^h(\lambda,j)}\right)\left(\epsilon(z,j)^{-\theta(\lambda)}\frac{1}{\pi^h(\lambda,j)}\right)^{-\frac{1}{\theta(\lambda)}}\partial r$$

$$= -\int_0^\infty \exp\left(-\epsilon(z,j)^{-\theta(\lambda)}\frac{1}{\pi^h(\lambda,j)}\right)\epsilon(z,j)\pi^h(\lambda,j)^{\frac{1}{\theta(\lambda)}-1}\left(-\theta(\lambda)\right)\epsilon(z,j)^{-\theta(\lambda)-1}\partial\epsilon$$

$$A^h(\lambda)T(\lambda,j)\boldsymbol{E}\left(\epsilon_z|z\in Z_h(\lambda), w_z(j)\geq w_z(j')\,\forall j'\in J\right)$$

$$= A^h(\lambda)T(\lambda,j)\frac{1}{\pi^h(\lambda,j)}\Gamma(1-\frac{1}{\theta(\lambda)})\pi^h(\lambda,j)^{1-\frac{1}{\theta(\lambda)}} = A^h(\lambda)T(\lambda,j)\Gamma(1-\frac{1}{\theta(\lambda)})\pi^h(\lambda,j)^{-\frac{1}{\theta(\lambda)}}$$

$$Total\, supply\, of\, good\, (j,h)\, is: \quad \textstyle\sum_\lambda A^h(\lambda)T(\lambda,j)\Gamma(1-\frac{1}{\theta(\lambda)})\pi^h(\lambda,j)^{-\frac{1}{\theta(\lambda)}}L^h(\lambda)\pi^h(\lambda,j)$$

$$= \sum_\lambda A^h(\lambda)T(\lambda,j)\Gamma(1-\frac{1}{\theta(\lambda)})\pi^h(\lambda,j)^{1-\frac{1}{\theta(\lambda)}}L^h(\lambda)$$

9.5 Gauss-Jacobi Algorithm and Property of the Equilibrium

The Gauss-Jacobi algorithm procedure reduces the problem of solving for n unknowns simultaneously in n equations to that of repeatly solving n equations with one unknown. More specifically, given the known value of the kth iterate, x^k, one uses the ith equation to compute the ith component of unknown x^{k+1}, the next iterate. Formally x^{k+1} is defined in terms of x^k by the following equations:

$$\begin{aligned} f^1(x_1^{k+1}, x_2^k, x_3^k, ..., x_n^k) &= 0 \\ f^2(x_1^k, x_2^{k+1}, x_3^k, ..., x_n^k) &= 0 \\ &... \\ f^n(x_1^k, x_2^k, ..., x_{n-1}^k, x_n^{k+1}) &= 0 \end{aligned}$$

The linear Gauss-Jacobi method takes a single Newton step to approximate the components of x^{k+1}. The resulting scheme is $x_i^{k+1} = x_i^k - \frac{f^i(x^k)}{f_{xi}^i(x^k)}$, $i = 1, ..., n$.

Note that the set of prices enter both the demand side and the supply side nonlinearly. In general, for a system of nonlinear equations, it is not possible to characterize the conditions under which a solution exists or is unique. I appeal to the Implicit Function Theorem to show that the price equilibrium that we have found numerically using the Gauss-Jacobi method is locally isolated as a function of the parameters. It states that if F is continuously differentiable, $F(x^*) = 0$ and $DF(x^*)$ has full rank, then the zero set of F is, near x^*, an N-dimensional surface in R^L. My excess demand functions are continuously differentiable and the vector of prices set them to 0. Also, the Jacobian matrix of these functions has full rank ($J * N = 1400$).

Total Derivative Applying the Gauss-Jacobi method to numerically calcuate the set of equilibrium prices requires computing the total derive of the market clearing conditions with respect to the prices. This section provides more detail.

Supply Recall that the total supply of good (j, h) is:

$$\sum_{\lambda} A^h(\lambda) T(\lambda, j) \Gamma(1 - \frac{1}{\theta(\lambda)}) \pi^h(\lambda, j)^{1-\frac{1}{\theta(\lambda)}} L^h(\lambda)$$

Totally differentiate this term, I have:

$$\sum_{\lambda} A^h(\lambda)T(\lambda,j)L^h(\lambda)\Gamma\left(1-\frac{1}{\theta(\lambda)}\right)\left(1-\frac{1}{\theta(\lambda)}\right)\pi^h(\lambda,j)^{-\frac{1}{\theta(\lambda)}}$$

$$\left[\frac{\partial\pi^h(\lambda,j)}{\partial p^h_{(j,h)}}\partial p^h_{(j,h)}+\sum_{j'\neq j}\frac{\partial\pi^h(\lambda,j)}{\partial p^h_{(j',h)}}\partial p^h_{(j',h)}\right]$$

$$\pi^h\left(\lambda,j\right) = \frac{\left[p^h_{(j,h)}A^h(\lambda)T\left(\lambda,j\right)\right]^{\theta(\lambda)}}{\sum_{j'\in\mathcal{J}}\left[p^h_{(j',h)}A^h(\lambda)T\left(\lambda,j'\right)\right]^{\theta(\lambda)}}$$

$$\frac{\partial\pi^h(\lambda,j)}{\partial p^h_{(j,h)}} = \frac{\theta(\lambda)\left[p^h_{(j,h)}A^h(\lambda)T(\lambda,j)\right]^{\theta(\lambda)-1}A^h(\lambda)T(\lambda,j)}{\sum_{j'\in\mathcal{J}}\left[p^h_{(j',h)}A^h(\lambda)T\left(\lambda,j'\right)\right]^{\theta(\lambda)}}$$

$$-\frac{\left[p^h_{(j,h)}A^h(\lambda)T\left(\lambda,j\right)\right]^{\theta(\lambda)}\theta(\lambda)\left[p^h_{(j,h)}A^h(\lambda)T(\lambda,j)\right]^{\theta(\lambda)-1}A^h(\lambda)T(\lambda,j)}{\left\{\sum_{j'\in\mathcal{J}}\left[p^h_{(j',h)}A^h(\lambda)T\left(\lambda,j'\right)\right]^{\theta(\lambda)}\right\}^2}$$

$$\frac{\partial\pi^h(\lambda,j)}{\partial p^h_{(j,h)}} = \frac{\theta(\lambda)\left[A^h(\lambda)T(\lambda,j)\right]^{\theta(\lambda)}\left(p^h_{(j,h)}\right)^{\theta(\lambda)-1}}{\sum_{j'\in\mathcal{J}}\left[p^h_{(j',h)}A^h(\lambda)T\left(\lambda,j'\right)\right]^{\theta(\lambda)}}$$

$$-\frac{\theta(\lambda)\left[A^h(\lambda)T(\lambda,j)\right]^{2\theta(\lambda)}\left(p^h_{(j,h)}\right)^{2\theta(\lambda)-1}}{\left\{\sum_{j'\in\mathcal{J}}\left[p^h_{(j',h)}A^h(\lambda)T\left(\lambda,j'\right)\right]^{\theta(\lambda)}\right\}^2} = \theta(\lambda)\left[A^h(\lambda)^{\theta(\lambda)}T(\lambda,j)^{\theta(\lambda)}\left(p^h_{(j,h)}\right)^{\theta(\lambda)-1}\right]*$$

$$\frac{\left\{\sum_{j'\in\mathcal{J}}\left[p^h_{(j',h)}A^h(\lambda)T\left(\lambda,j'\right)\right]^{\theta(\lambda)}-\left[p^h_{(j,h)}A^h(\lambda)T\left(\lambda,j\right)\right]^{\theta(\lambda)}\right\}}{\left\{\sum_{j'\in\mathcal{J}}\left[p^h_{(j',h)}A^h(\lambda)T\left(\lambda,j'\right)\right]^{\theta(\lambda)}\right\}^2}$$

$$=\frac{\theta(\lambda)\left[A^h(\lambda)T(\lambda,j)\right]^{\theta(\lambda)}\left(p^h_{(j,h)}\right)^{\theta(\lambda)-1}\left[\sum_{j'\neq j}\left[p^h_{(j',h)}A^h(\lambda)T\left(\lambda,j'\right)\right]^{\theta(\lambda)}\right]}{\left\{\sum_{j'\in\mathcal{J}}\left[p^h_{(j',h)}A^h(\lambda)T\left(\lambda,j'\right)\right]^{\theta(\lambda)}\right\}^2}$$

$$\frac{\partial\pi^h(\lambda,j)}{\partial p^h_{(j',h)}} = -\frac{\left[p^h_{(j,h)}A^h(\lambda)T\left(\lambda,j\right)\right]^{\theta(\lambda)}\theta(\lambda)\left[p^h_{(j',h)}A^h(\lambda)T\left(\lambda,j'\right)\right]^{\theta(\lambda)-1}A^h(\lambda)T(\lambda,j')}{\left\{\sum_{j'\in\mathcal{J}}\left[p^h_{(j',h)}A^h(\lambda)T\left(\lambda,j'\right)\right]^{\theta(\lambda)}\right\}^2}$$

$$=-\frac{\left[p^h_{(j,h)}A^h(\lambda)T\left(\lambda,j\right)\right]^{\theta(\lambda)}\theta(\lambda)\left[A^h(\lambda)T\left(\lambda,j'\right)\right]^{\theta(\lambda)}\left(p^h_{(j',h)}\right)^{\theta(\lambda)-1}}{\left\{\sum_{j'\in\mathcal{J}}\left[p^h_{(j',h)}A^h(\lambda)T\left(\lambda,j'\right)\right]^{\theta(\lambda)}\right\}^2}$$

$$\frac{\partial\pi^h(\lambda,j)}{\partial p^h_{(j,h)}}\partial p^h_{(j,h)}+\sum_{j'\neq j}\frac{\partial\pi^h(\lambda,j)}{\partial p^h_{(j',h)}}\partial p^h_{(j',h)} = -\frac{numerator}{\left\{\sum_{j'\in\mathcal{J}}\left[p^h_{(j',h)}A^h(\lambda)T\left(\lambda,j'\right)\right]^{\theta(\lambda)}\right\}^2}$$

$$numerator=\theta(\lambda)\left[p^h_{(j,h)}A^h(\lambda)T\left(\lambda,j\right)\right]^{\theta(\lambda)}\left\{\hat{p}^h_{(j,h)}\sum_{j'\neq j}\left[p^h_{(j,h)}A^h(\lambda)T(\lambda,j')\right]^{\theta(\lambda)}\right.$$

$$\left.-\sum_{j'\neq j}\left[A^h(\lambda)T(\lambda,j')\right]^{\theta(\lambda)}(p^h_{(j',h)})^{\theta(\lambda)}\hat{p}^h_{(j',h)}\right\}$$

Demand The total demand for good (j,h) is $\frac{\sum_n S^n_{(j,h)}\bar{w}^n L^n}{p^h_{(j,h)}}$. Totally differentiate this term, I have:

$$-\frac{\sum_n S^n_{(j,h)} \bar{w}^n L^n}{\left(p^h_{(j,h)}\right)^2} \partial p^h_{(j,h)} + \frac{\sum_n \partial S^n_{(j,h)} \bar{w}^n L^n + \sum_n S^n_{(j,h)} L^n \partial \bar{w}^n}{p^h_{(j,h)}}$$

where $\partial S^n_{(j,h)}$ and $\partial \bar{w}^n$ are total differentiation of $S^n_{(j,h)}$ and $\bar{w}^n$ with respect to prices.

Under the parametric restrictions, aggregate expenditure shares can be simplified to:

$$S^n_{(j,h)} = \alpha^n_{(j,h)} - \gamma_j \ln p^n_{(j,h)} + \frac{\gamma_j}{N} \sum_{n'=1}^{N} \ln p^n_{(j,n')} + \beta_{(j,h)} y^n$$

Totally differentiate $S^n_{(j,h)}$, I obtain:

$$\begin{aligned} \partial S^n_{(j,h)} &= -\gamma_j \hat{p}^n_{(j,h)} + \frac{\gamma_j}{N} \sum_{n'=1}^{N} \hat{p}^n_{(j,n')} + \beta_{(j,h)} \partial y^n \\ &= -\gamma_j \left(\hat{p}^h_{(j,h)} + \hat{\tau}^n_{(j,h)}\right) + \frac{\gamma_j}{N} \sum_{n'=1}^{N} \left(\hat{p}^{n'}_{(j,n')} + \hat{\tau}^n_{(j,n')}\right) + \beta_{(j,h)} \partial y^n \end{aligned}$$

The unadjusted average wage $\bar{w}^n = \sum_\lambda \frac{L^n(\lambda)}{L^n} \Gamma(\lambda) x^n(\lambda)$ where

$$x^n(\lambda) \equiv \left(\sum_j \left[p^n_{(j,n)} A^n(\lambda) T(\lambda, j) \right]^{\theta(\lambda)} \right)^{\frac{1}{\theta(\lambda)}}$$

$$\partial \bar{w}^n = \sum_\lambda \frac{L^n(\lambda)}{L^n} \Gamma(\lambda) \frac{1}{\theta(\lambda)} \left(\sum_j \left[p^n_{(j,n)} A^n(\lambda) T(\lambda, j) \right]^{\theta(\lambda)} \right)^{\frac{1}{\theta(\lambda)}-1}$$

$$\sum_j \left\{ \theta(\lambda) \left[p^n_{(j,n)} A^n(\lambda) T(\lambda, j) \right]^{\theta(\lambda)-1} A^n(\lambda) T(\lambda, j) \partial p^n_{(j,n)} \right\}$$

$$\partial \bar{w}^n = \sum_\lambda \frac{L^n(\lambda)}{L^n} \Gamma(\lambda) \frac{1}{\theta(\lambda)} \left(\sum_j \left[p^n_{(j,n)} A^n(\lambda) T(\lambda, j) \right]^{\theta(\lambda)} \right)^{\frac{1-\theta(\lambda)}{\theta(\lambda)}}$$

$$\sum_j \left\{ \theta(\lambda) \left[p^n_{(j,n)} A^n(\lambda) T(\lambda, j) \right]^{\theta(\lambda)} \hat{p}^n_{(j,n)} \right\}$$

$$= \sum_{\lambda} \frac{L^n(\lambda)}{L^n} \Gamma(\lambda) x^n(\lambda) \left[\sum_j \pi^n(\lambda, j) \hat{p}^n_{(j,n)} \right]$$

Alternatively, $\partial \bar{w}^n = \sum_{\lambda} \frac{L^n(\lambda)}{L^n} \Gamma(\lambda) \partial x^n$ where

$$\partial x^n = \frac{1}{\theta(\lambda)} \left(\sum_j \left[p^n_{(j,n)} A^n(\lambda) T(\lambda, j) \right]^{\theta(\lambda)} \right)^{\frac{1}{\theta(\lambda)} - 1}$$

$$\sum_j \theta(\lambda) \left[p^n_{(j,h)} A^n(\lambda) T(\lambda, j) \right]^{\theta(\lambda) - 1} A^n(\lambda) T(\lambda, j) \partial p^n_{(j,n)}$$

$$= x^n(\lambda) \left[\sum_j \pi^n(\lambda, j) \hat{p}^n_{(j,n)} \right]$$

$$\partial \bar{w}^n = \sum_{\lambda} \frac{L^n(\lambda)}{L^n} \Gamma(\lambda) x^n(\lambda) \left[\sum_j \pi^n(\lambda, j) \hat{p}^n_{(j,n)} \right]$$

∂y^n

$$y^n = \ln \left(\frac{\tilde{w}^n}{a(p^n)} \right) = \ln \left(\frac{\tilde{w}^n}{a^n} \right) \quad \rightarrow \quad \partial y^n = \partial \ln \tilde{w}^n - \partial \ln a^n = \hat{\tilde{w}}^n - \hat{a}^n$$

$$\ln a^n = \underline{\alpha} + \textstyle\sum_{(j,h)} \alpha^n_{(j,h)} \ln p^n_{(j,h)} + \sum_{(j,h)} \sum_{(j',h')} \gamma_{(j,h)(j',h')} \ln p^n_{(j,h)} \ln p^n_{(j',h')}$$

$$\hat{a}^n = \frac{\partial \ln a}{\partial \ln p^n} \hat{p}^n \quad = \quad \sum_j \sum_h \left[\alpha^n_{(j,h)} + \frac{1}{2} * \frac{-2(N-1)}{N} \gamma_j \ln p^n_{(j,h)} \right.$$

$$\left. + \frac{1}{2} \sum_{j'=j, h' \neq h} \frac{\gamma_j}{N} \ln p^n_{(j',h')} + \frac{1}{2} \sum_{j' \neq j, h' \neq h} \frac{\gamma_j}{N} \ln p^n_{(j',h')} \right] \hat{p}^n_{(j,h)}$$

Under the restrictions on the matrix Γ,

$$\hat{a}^n = \sum_j \sum_h \left[\alpha^n_{(j,h)} - \frac{N-1}{N} \gamma_j \ln p^n_{(j,h)} + \sum_{j'=j, h' \neq h} \frac{\gamma_j}{N} \ln p^n_{(j',h')} \right] \hat{p}^n_{(j,h)}$$

Rewrite,

$$\hat{a}^n = \sum_j \sum_h \left[\alpha^n_{(j,h)} - \gamma_j \ln p^n_{(j,h)} + \tfrac{\gamma_j}{N} \sum_{h'=1}^N \ln p^n_{(j,h')} \right] \hat{p}^n_{(j,h)}$$

The term inside the bracket is $S^n_{(j,h)} - \beta_{(j,h)} y^n$. Therefore,

$$\partial y^n = \hat{\tilde{w}}^n - \hat{a}^n = \hat{\tilde{w}}^n - \sum_j \sum_h \left[S^n_{(j,h)} - \beta_{(j,h)} y^n \right] \left(\hat{\tau}^n_{(j,h)} + \hat{p}^h_{(j,h)} \right))$$

The change in the inequality-adjusted average income can be expressed in terms of changes in log prices:

$$\begin{aligned} \hat{\tilde{w}}^n &= \hat{\bar{w}}^n + \partial \Sigma^n \\ &= -\left(\frac{1}{\bar{w}^n} \right)^2 \left[\sum_\lambda \frac{L^n(\lambda)}{L^n} \Gamma(\lambda) \left(\ln x^n(\lambda) - \frac{\Psi(\lambda)}{\tilde{\theta}(\lambda)} \right) x^n(\lambda) \right] \partial \bar{w}^n \\ &\quad + \frac{1}{\bar{w}^n} \sum_\lambda \frac{L^n(\lambda)}{L^n} \Gamma(\lambda) \left(1 + \ln x^n(\lambda) - \frac{\Psi(\lambda)}{\tilde{\theta}(\lambda)} \right) \partial x^n(\lambda) \end{aligned}$$

where

$$\begin{aligned} \partial \bar{w}^n &= \sum_\lambda \frac{L^n(\lambda)}{L^n} \Gamma(\lambda) x^n(\lambda) \left[\sum_j \pi^n(\lambda, j) \hat{p}^n_{(j,n)} \right] \\ \partial x^n(\lambda) &= x^n(\lambda) \left[\sum_j \pi^n(\lambda, j) \hat{p}^n_{(j,n)} \right] \end{aligned}$$

The unadjusted average wage and the Theil index can be expressed in terms of $x^n(\lambda)$:

$$\begin{aligned} \bar{w}^n &= \sum_\lambda \frac{L^n(\lambda)}{L^n} \Gamma(\lambda) x^n(\lambda) \\ \Sigma^n &= \frac{1}{\bar{w}^n} \sum_\lambda \frac{L^n(\lambda)}{L^n} \Gamma(\lambda) \left(x^n(\lambda) \ln x^n(\lambda) - \frac{\Psi(\lambda)}{\tilde{\theta}(\lambda)} x^n(\lambda) \right) - \ln \bar{w}^n \end{aligned}$$

where $x^n(\lambda) \equiv \left(\sum_{j' \in \mathcal{J}} \left[p^h_{(j',h)} A^h(\lambda) T(\lambda, j') \right]^{\theta(\lambda)} \right)^{\frac{1}{\theta(\lambda)}}$. $\Gamma(\lambda) \equiv \Gamma\left(1 - \frac{1}{\tilde{\theta}(\lambda)}\right)$ is the gamma function and $\Psi(\lambda) \equiv \Psi\left(1-\frac{1}{\tilde{\theta}(\lambda)}\right)$ is the digamma function.

Finally, $S^n_{(j,h)}$ is a function of $\{\ln p^n_{(j,h)}\}_{h\in N}$, that is, $\{\ln p^h_{(j,h)}\}_{h\in N}$ and $\{\ln \tau^n_{(j,h)}\}_{h\in N}$. $y^n = \ln \bar{w}^n + \Sigma^n - \ln a(p^n)$ is a function of $\{p^h_{(j,h)}\}_{j\in J}$, $\{\ln p^h_{(j,h)}\}_{h\in N}$ and $\{\ln \tau^n_{(j,h)}\}_{h\in N}$.

9.6 Countries in IPUMS-I

Brazil (2000), Canada (2001), Colombia (1973), India (2004), Jamaica (2001), Mexico (2000), Panama (2000), United States (2005), Uruguay (2006), Venezuela (2001), Israel (1995), Germany (1970), Puerto Rico (2005), Indonesia (1995), South Africa (2007), Dominican Republic (2002).

9.7 Absolute Advantage $A^h(\lambda)$

$$
\begin{aligned}
x^h(\lambda) = \left(\sum_j x^h(\lambda,j)^{\theta(\lambda)}\right)^{\frac{1}{\theta(\lambda)}} &= \left\{\sum_{j\in\mathcal{J}} \left[p^h_{(j,h)} A^h(\lambda) T(\lambda,j)\right]^{\theta(\lambda)}\right\}^{\frac{1}{\theta(\lambda)}} \\
&= A^h(\lambda)\left\{\sum_{j\in\mathcal{J}} \left[p^h_{(j,h)} T(\lambda,j)\right]^{\theta(\lambda)}\right\}^{\frac{1}{\theta(\lambda)}}
\end{aligned}
$$

$$
log\, x^h(\lambda) = log\, A^h(\lambda) + \frac{1}{\theta(\lambda)} log \left\{\sum_{j\in\mathcal{J}} \left[p^h_{(j,h)} T(\lambda,j)\right]^{\theta(\lambda)}\right\}
$$

Take a first-order approximation at $\boldsymbol{p}=\mathbf{1}$, $\boldsymbol{T}=\mathbf{1}$:

$$
\begin{aligned}
log\, x^h(\lambda) &= log\, A^h(\lambda) + \frac{1}{\theta(\lambda)}\left\{log\, J + \frac{1}{J}\sum_{j\in\mathcal{J}}\left(\left[p^h_{(j,h)} T(\lambda,j)\right]^{\theta(\lambda)} - 1\right)\right\} \\
&= log\, A^h(\lambda) + \frac{1}{\theta(\lambda)}\left\{log\, J + \frac{1}{J}\sum_{j\in\mathcal{J}} log\left(\left[p^h_{(j,h)} T(\lambda,j)\right]^{\theta(\lambda)}\right)\right\} \\
&= log\, A^h(\lambda) + \frac{1}{\theta(\lambda)} log\, J + \frac{1}{J}\left[\sum_{j\in\mathcal{J}} log\, p^h_{(j,h)} + \sum_{j\in J} log\, T(\lambda,j)\right]\Bigg\}
\end{aligned}
$$

$$
log\, x^h(1) = log\, A^h(1) + \frac{1}{\theta(1)} log\, J + \frac{1}{J}\left[\sum_{j\in\mathcal{J}} log\, p^h_{(j,h)} + \sum_{j\in J} log\, T(1,j)\right]\Bigg\}
$$

$$log\left(\frac{x^h(\lambda)}{x^h(1)}\right) = log\left(\frac{A^h(\lambda)}{A^h(1)}\right) + log\, J\left(\frac{1}{\theta(\lambda)} - \frac{1}{\theta(1)}\right) + \frac{1}{J}\sum_{j\in J} log\left(\frac{T(\lambda,j)}{T(1,j)}\right)$$

9.8 Labor Groups

labor group	1	2	3	4	5	6	7	8	9
sex	Male	Male	Male	Male	Male	Male	Male	Male	Male
age	15-24	15-24	15-24	25-49	25-49	25-49	50-74	50-74	50-74
edu	ED0-2	ED3-4	ED5-8	ED0-2	ED3-4	ED5-8	ED0-2	ED3-4	ED5-8
labor group	10	11	12	13	14	15	16	17	18
sex	Female	Female	Female	Female	Female	Female	Female	Female	Female
age	15-24	15-24	15-24	25-49	25-49	25-49	50-74	50-74	50-74
edu	ED0-2	ED3-4	ED5-8	ED0-2	ED3-4	ED5-8	ED0-2	ED3-4	ED5-8

Table 6: Labor Groups

9.9 Non-homothetic Gravity Equation

Under the additional assumptions on $\mathbf{\Gamma}$,

$$\frac{Y^h_{(j,n)}}{Y^h} \equiv S^h_{(j,n)} = \alpha^h_{(j,n)} - \gamma_j \ln\left(\frac{p^h_{(j,n)}}{P^h_j}\right) + \beta_{(j,n)} y^h$$

where $P^h_j = \exp\left(\frac{1}{N}\sum_{n'} \ln p^h_{(j,n')}\right)$. Replacing $p^h_{(j,n')} = \tau^h_{(j,n')} p^{n'}_{(j,n')}$, I have:

$$\frac{p^h_{(j,n)}}{P^h_j} = \frac{\tau^h_{(j,n)}}{\exp\left(\frac{1}{N}\sum_{n'} \ln \tau^h_{(j,n')}\right)} \cdot \frac{p^n_{(j,n)}}{\exp\left(\frac{1}{N}\sum_{n'} \ln p^{n'}_{(j,n')}\right)} \equiv \frac{\tau^h_{(j,n)}}{\bar{\tau}^h_j} \cdot \frac{p^n_{(j,n)}}{\bar{p}_j}$$

Therefore,

$$\begin{aligned} \frac{Y_{(j,n)}}{Y^W} &= \sum_{n'} \frac{Y^{n'}}{Y^W} S^{n'}_{(j,n)} \\ &= \sum_{n'} \frac{Y^{n'}}{Y^W}\left(\alpha^{n'}_{(j,n)} - \gamma_j \ln\left(\frac{\tau^{n'}_{(j,n)}}{\bar{\tau}^{n'}_j} \cdot \frac{p^n_{(j,n)}}{\bar{p}_j}\right) + \beta_{(j,n)} y^{n'}\right) \end{aligned}$$

Substract the second equation from the first,

$$\frac{Y^h_{(j,n)}}{Y^h} - \frac{Y_{(j,n)}}{Y^W} = \underbrace{\left[\alpha^h_{(j,n)} - \sum_{n'} \frac{Y^{n'}}{Y^W} \alpha^{n'}_{(j,n)}\right]}_{\equiv K^h_{(j,n)}}$$

$$-\gamma_j \underbrace{\left[\ln\left(\frac{\tau^h_{(j,n)}}{\bar{\tau}^h_j} \cdot \frac{p^n_{(j,n)}}{\bar{p}_j}\right) - \sum_{n'} \frac{Y^{n'}}{Y^W} \ln\left(\frac{\tau^{n'}_{(j,n)}}{\bar{\tau}^{n'}_j} \cdot \frac{p^n_{(j,n)}}{\bar{p}_j}\right)\right]}_{\equiv M^h_{(j,n)}}$$

$$+\beta_{(j,n)} \underbrace{\left[y^h - \sum_{n'} \frac{Y^{n'}}{Y^W} y^{n'}\right]}_{\equiv \Omega^h}$$

9.10 The Differences in Tastes across Countries

Under the additional assumptions on $\mathbf{\Gamma}$ and $\sum_n \alpha_n = 1$ combined with the equation $\alpha^h_{(j,n)} = \alpha_n(\alpha_j + \epsilon^h_j)$,

$$S^h_j = \sum_n \alpha^h_{(j,n)} + \bar{\beta}_j y^h = \alpha_j + \bar{\beta}_j y^h + \epsilon^h_j$$

$$S^W_j = \frac{Y^W_j}{Y^W} = \frac{\sum_{n'=1}^N Y^{n'} S^{n'}_j}{Y^W} = \sum_{n'=1}^N \frac{Y^{n'}}{Y^W}\left(\alpha_j + \bar{\beta}_j y^h + \epsilon^h_j\right)$$

$$S^h_j - S^W_j = \alpha_j - \sum_{n'=1}^N \left(\frac{Y^{n'}}{Y^W}\right)\alpha_j + \epsilon^h_j - \sum_{n'=1}^N \left(\frac{Y^{n'}}{Y^W}\right)\epsilon^h_j + \bar{\beta}_j \Omega^h$$

$$K^h_{(j,n)} = \alpha^h_{(j,n)} - \sum_{n'=1}^N \left(\frac{Y^{n'}}{Y^W}\right)\alpha^{n'}_{(j,n)} = \alpha_n(\alpha_j + \epsilon^h_j) - \sum_{n'=1}^N \left(\frac{Y^{n'}}{Y^W}\right)\alpha_n(\alpha_j + \epsilon^{n'}_j)$$
$$= \alpha_n\left[\alpha_j - \sum_{n'=1}^N \left(\frac{Y^{n'}}{Y^W}\right)\alpha_j + \epsilon^h_j - \sum_{n'=1}^N \left(\frac{Y^{n'}}{Y^W}\right)\epsilon^h_j\right] = \alpha_n(S^h_j - S^W_j) - \alpha_n \bar{\beta}_j \Omega^h$$

9.11 Counterfactual - Back to Autarky

Recall that the global welfare change of individual z under the AIDS between an initial scenario under trade and a counterfactual scenario is:

$$u_z^{tr\to cf} \quad = \quad (\frac{E_{cf}^h}{E_{tr}^h})(\frac{w_z^{tr}}{\tilde{w}_{tr}^h})^{-\ln(b_{cf}^h/b_{tr}^h)}(\frac{w_z^{cf}}{w_z^{tr}})$$

where E_{cf}^h/E_{tr}^h and $-\ln(b_{cf}^h/b_{tr}^h)$ are functions of the prices in the two scenarios. I need to compute the prices of domestic commodities in autarky, $\{p_{(j,h)}^{h,cf}\}$, as well as the consumer-specific reservation prices of foreign varieties, $\{p_{(j,n),z}^{h,cf}\}$, that are no longer consumed.

The restriction to non-negative individual expenditure shares may bind in the counterfactual. In these cases, I find consumer-specific reservation prices such that the individual shares of dropped varieties are all zero and the remaining individual shares are adjusted using these reservation prices. For each percentile z in country h, I have that $p_{(j,h),z}^{h,cf} = p_{(j,h)}^{h,cf}$ for all j and $s_{(j,n),z}^{h,cf} = 0$ for all j and $n \neq h$.

Reservation prices, $p_{(j,n),z}^{h,cf}$, for $n \neq h$ and individual shares, $s_{(j,h),z}^{h,cf}$, satisfy:

$$s_{(j,h),z}^{h,cf} = \alpha_{(j,h)}^h - \gamma_j \ln p_{(j,h)}^{h,cf} + \frac{\gamma_j}{N}\left(\ln p_{(j,h)}^{h,cf} + \sum_{n\neq h} \ln p_{(j,n),z}^{h,cf}\right) + \beta_{(j,n)} \ln\left(\frac{w_z^{cf}}{a_{h,z}^{cf}}\right)$$

$$0 = \alpha_{(j,n)}^h - \gamma_j \ln p_{(j,n),z}^{h,cf} + \frac{\gamma_j}{N}\left(\ln p_{(j,h)}^{h,cf} + \sum_{n\neq h} \ln p_{(j,n),z}^{h,cf}\right) + \beta_{(j,n)} \ln\left(\frac{w_z^{cf}}{a_{h,z}^{cf}}\right)$$

The second equation implies that:

$$\sum_{n\neq h} \gamma_j \ln p_{(j,n),z}^{h,cf} = (N-1)\ln p_{(j,h)}^{h,cf} + N\left(\sum_{n\neq h} \alpha_{(j,n)}^h + \sum_{n\neq h} \beta_{(j,n)} \ln\left(\frac{w_z^{cf}}{a_{h,z}^{cf}}\right)\right)$$

Replacing this back into the second equation gives the reservation prices of the foreign varieties in sector j:

$$\ln p_{(j,n),z}^{h,cf} = \frac{1}{\gamma_j}\left[\alpha_{(j,n)}^h + \sum_{n'\neq h} \alpha_{(j,n')}^h + \left(\beta_{(j,n)+} \sum_{n'\neq h} \beta_{(j,n')}\right) \ln\left(\frac{w_z^{cf}}{a_{h,z}^{cf}}\right)\right] + \ln\left(p_{(j,h)}^{h,cf}\right)$$

where $a_{h,z}^{cf} = a\left(\{p_{(j,h)}^{h,cf}, p_{(j,n),z}^{h,cf}\}\right)$ is the homothetic component of the price index, and w_z^{cf} is the autarky income of percentile z from the home country, and it is a function of $\{p_{(j,h)}^{h,cf}\}$. For each h, this gives me $(N-1)*J*Z$ equations in $\{p_{(j,h)}^{h,cf}, p_{(j,n),z}^{h,cf}\}$.

I combine these reservation price equations with J market clearing conditions in autarky, which equalize the total supply and the total demand.

The total supply of good (j,h) is:

$$\sum_{\lambda} A^h(\lambda)T(\lambda,j)\Gamma(1-\frac{1}{\theta(\lambda)})\pi^h(\lambda,j)^{1-\frac{1}{\theta(\lambda)}}L^h(\lambda)$$

which is a function of $\{p^{h,cf}_{(j,h)}\}$ in autarky.

Under the parametric restrictions, the expenditure share for consumer z on goods from country n in sector j is:

$$s^h_{(j,n),z} = \alpha^h_{(j,n)} - \gamma_j \ln p^h_{(j,n)} + \frac{\gamma_j}{N}\sum_{n'=1}^{N} \ln p^h_{(j,n')} + \beta_{(j,n)} \ln(\frac{w_z}{a(p^h)})$$

Adding up across n, the share of sector j in total expenditures of z is:

$$s^h_{j,z} = \sum_n s^h_{(j,n),z} = \bar{\alpha}^h_j + \bar{\beta}_j \ln(\frac{w_z}{a(p^h)})$$

where $\bar{\alpha}^h_j = \sum_n \alpha^h_{(j,n)}$ and $\bar{\beta}_j = \sum_n \beta_{(j,n)}$.

In autarky, the expenditure shares evaluated at the reservation prices are such that:

$$s^{h,cf}_{(j,h),z} = \bar{\alpha}^h_j + \bar{\beta}_j \ln\left(\frac{w^{cf}_z}{a^{cf}_{h,z}}\right)$$
$$s^{h,cf}_{(j,n),z} = 0,\ n \neq h$$

The aggregate expenditure shares are thus: $S^{h,cf}_{(j,h)} = \sum_z (\frac{w^{cf}_z}{\sum_{z'} w_{z'}}) s^{h,cf}_{(j,h),z}$.

Since $\bar{w}^{cf}_h$ is a function of $\{p^{h,cf}_{(j,h)}\}$, the total demand for good (j,h) in autarky, $S^{h,cf}_{(j,h)}\bar{w}^{cf}_h L^h/p^{h,cf}_{(j,h)}$, is a function of $\{p^{h,cf}_{(j,h)}, p^{h,cf}_{(j,n),z}\}$. Market clearing conditions give me J equations in $\{p^{h,cf}_{(j,h)}, p^{h,cf}_{(j,n),z}\}$ for each h. Combined with the equations of reservation prices above, I have $J+(N-1)*J*Z$ equations in $J+(N-1)*J*Z$ unknowns.

Once I solve for the output prices, $\{p^{h,cf}_{(j,h)}\}_{j\in J}$, in autarky, I can back out $w^h(\lambda)$, and therefore, the wage distribution for each labor type λ:

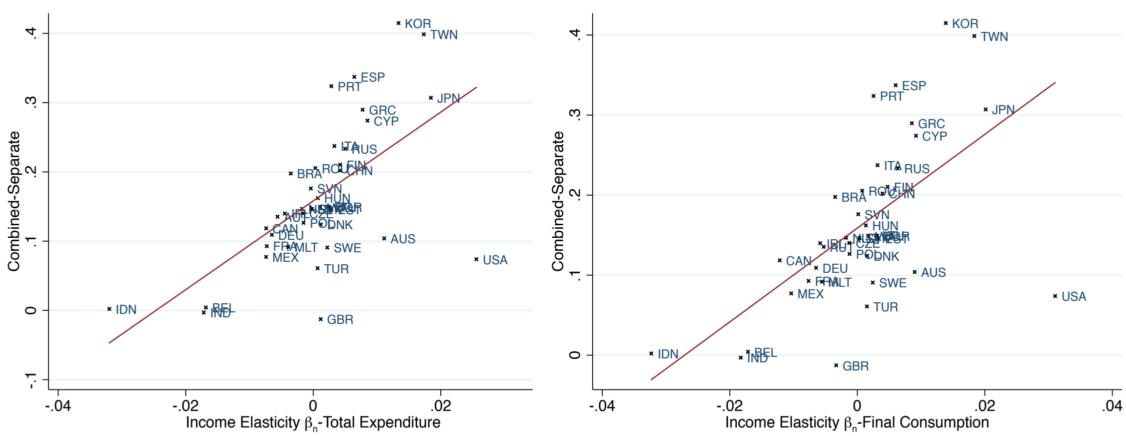
$$x^h(\lambda) = \left(\sum_j \left[A^h(\lambda)p^h_{(j,h)}T(\lambda,j)\right]^{\theta(\lambda)}\right)^{\frac{1}{\theta(\lambda)}}$$

The autarky wage distribution is:

$$
\begin{aligned}
Pr(w_z \leq w) &= \sum_{\lambda} Pr\Bigg(w_z \leq w | z \in \mathcal{Z}^h(\lambda) \Bigg) Pr\Bigg(z \in \mathcal{Z}^h(\lambda) \Bigg) \\
&= \sum_{\lambda} \frac{L^h(\lambda)}{L^h} \exp\Bigg\{ -x^h(\lambda)^{\theta(\lambda)} w^{-\theta(\lambda)} \Bigg\} = \sum_{\lambda} \exp\Bigg\{ -x^h(\lambda)^{\theta(\lambda)} w^{-\theta(\lambda)} + \ln\Bigg(\frac{L^h(\lambda)}{L^h} \Bigg) \Bigg\}
\end{aligned}
$$

9.12 Bias and Income Elasticity

Figure 16 plots the difference in the poor's relative benefit from trade liberalization between estimating the two effects jointly and separately against the income elasticity of the country's production, $\bar{\beta}^h_{prod} = \sum_j \beta_{(j,h)}$. Panel A uses the income elasticity computed from total expenditure while Panel B is restricted to final consumption. The correlation between the bias and the income elasticity of a country's production remains positive and significant after excluding Luxembourg. This implies that the interaction of the two channels benefits more the countries that produce high-income elastic goods.



$Correlation = 0.6163$ $\qquad$ $Correlation = 0.6126$

Figure 16: Underprediction of Pro-poor Bias of Trade Liberalization