Industry Linkages from Joint Production

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Abstract

Does joint production within firms link up aggregate outcomes across industries? I exploit exogenous variation in foreign demand faced by US multi-industry manufacturers to identify this transmission mechanism. I find that a positive demand shock in one industry of a firm increases its sales in another the more that both industries share *knowledge* inputs—including R&D, IT, and other professional services. I develop a general equilibrium model of joint production and estimate that properties of knowledge inputs generate economies of scale and scope within the firm. An expansion of market size in one industry lowers prices in not only the same industry but also others. These cross-industry spillovers account for 20 percent of the aggregate response of prices to market size and manifest disproportionately among knowledge-intensive industries.

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Introduction

Multi-industry firms account for three-quarters of US manufacturing output. Do these firms operate independent industry-level production functions, or is output jointly determined across industries? Despite decades of intellectual history on *joint production* (Shephard, 1953; Baumol et al., 1982; Milgrom and Roberts, 1990), systematic firm-level evidence remains scarce. Workhorse models of multi-output firms (Klette and Kortum, 2004; Bernard et al., 2010; Mayer et al., 2014) assume instead that a firm's marginal cost in one industry is unaffected by its output in any other industry. In the absence of joint production, these models preclude within-firm cost complementarities from linking up aggregate outcomes across industries.

In this paper I provide firm-level evidence on joint production and a theoretical framework to quantify the new industry linkages that emerge. I find that a firm's sales in one industry responds to demand shocks in its other industries. To account for these intra-firm spillovers, I develop and estimate a tractable model of joint production featuring variable own and cross-industry price elasticities of supply. Under the estimated elasticities, joint production generates aggregate *economies of scope*—producer prices in one industry fall in response to expansions of market size in another industry. I calculate that economies of scope in US manufacturing account for 20 percent of the aggregate response of producer prices to market size and vary substantially across industries. I lay out these contributions over four sections of the paper.

Section 1 begins with an empirical test of production independence. I assemble panel data from the US Economic Census on the sales and exports, by industry, of all US manufacturing firms. I estimate the impact of plausibly exogenous demand shocks (originating from foreign buyers) in one industry of a firm on its sales in another industry. If production were independent, demand shocks would increase output in the directly affected industry but not other industries of the firm.

Instead, cross-industry spillovers are heterogeneous and depend on the types of inputs used in production. I measure industry-level expenditure shares across the universe of input categories in the BEA's input-output (I/O) and capital flow tables. I focus on a large but often-overlooked category of *knowledge-generating* inputs—comprised of professional services, information, and the leasing of intangible assets—that contribute to the firm's knowledge, or *intangible*, capital.[1] Despite the fact that 9 percent of gross manufacturing output is spent on this category of knowledge inputs, little is known about their role in

[1]The data covers both capitalized and current expenses over inputs such as R&D, engineering, management, IT and software. See Table 7 for a full list. McGrattan (2017) uses a similar classification in the BEA tables to estimate a multi-sector RBC model with intangible capital.

the production process.

I find that a demand shock in one industry of a firm increases its sales in another the more that the two industries share knowledge inputs. In contrast, spillovers are uncorrelated with the degree to which industries share other types of inputs. Relative to a broad literature documenting intra-firm spillovers (e.g. across plants in Giroud and Mueller, 2019), I provide the first evidence that transmission is heterogenous by industry and related to properties of knowledge inputs in production.[2] These findings are consistent with recent evidence that intangible capital might be scalable (Haskel and Westlake, 2017) and transferrable across the plants of a firm (Atalay et al., 2014).

Section 2 rationalizes intra-firm spillovers using a quantitative model of joint production. I develop a two-stage production framework where firms compete across multiple industries under monopolistic competition. In the first stage, the firm invests in a range of *capital* inputs to accumulate capital (both tangible and intangible) across industries. In the second stage, conditional on its accumulated levels of capital, the firm uses a range of *production* inputs to generate final output in each industry. Whereas the second stage is straightforward and separable across industries, the first stage is plausibly interdependent across industries.

I model capital accumulation as a stochastic process that reflects uncertainty behind the value of capital inputs, be it R&D, advertising, or land development. Each type of input contributes to an arrival process of new capital within the firm, and the firm allocates each new piece of capital to a chosen industry. This mapping from capital inputs to accumulated capital depends on two properties: scalability and mutability. Scalability measures the responsiveness of the capital arrival rate to increased input spending, and mutability determines the allocation of new capital across industries.

Mutability is a new concept relevant under joint production. Each new piece of capital arrives with a stochastic match-specific value in each potential industry. Mutability measures the variance in these match-specific values of capital conditional on arrival. A firm benefits from input mutability because it is able to choose the best industry outlet among multiple ex-post realizations of match-specific capital values. More mutable inputs are thus worth more in expectation, and more likely to contribute to capital accumulation across different industries.

Consider, for example, the R&D process within General Electric. An increase in demand in the aerospace industry would cause GE to scale up spending on materials science

[2]Other dimensions through which intra-firm spillovers occur include firm segments due to internal capital markets (Lamont, 1997), national boundaries due to knowledge transfer within multinationals (Keller and Yeaple, 2009; Desai et al., 2009; Bilir and Morales, 2019; Boehm et al., 2019a), market destinations due to convex production costs (Almunia et al., 2018), and distance (Giroud, 2013; Gumpert et al., 2019).

(a type of capital input), ostensibly to improve the hot gas path of its aviation turbines. However, the more mutable is scientific research, the more likely are the additional scientists to create inventions more valuable when deployed in GE's MRI machines (e.g. high-sensitivity scanning). The initial aerospace demand shock would thus increase GE's health equipment sales the more scalable and mutable is the production of scientific knowledge capital. These insights extend beyond R&D to all types of capital inputs, each of which can have varying degrees of scalability and mutability.[3]

The more that a pair of industries share scalable and mutable capital inputs, the greater are a firm's (quality-adjusted) production cost savings in one industry from an expansion of market size in another. Because different industries benefit from different types of capital, cross and own-industry price-elasticities of supply within the firm are heterogeneous. My quantitative framework links up an older literature on firm diversification and the production of knowledge (Penrose, 1959; Gort, 1962; Rubin, 1973; Griliches, 1979) with a newer one emphasizing firm heterogeneity in macro (Melitz, 2003). Whereas the former finds evidence of positive intra-firm R&D spillovers (Jovanovic, 1993; Klette, 1996), the latter focuses on non-supply-related mechanisms that range from demand cannibalization (Eckel and Neary, 2009; Dhingra, 2013), span-of-control (Nocke and Yeaple, 2014), to carry-along trade (Bernard et al., 2018). These models are typically solved within an industry equilibrium and feature symmetrically negative cross-product spillovers.[4]

In Section 3, I estimate the scalability and mutability of capital inputs in production across over 200 manufacturing industries. I leverage the intra-firm response of sales to demand shifters across industries—an identification insight often overlooked in the existing literature on joint production (Färe and Primont, 1995).[5] In response to demand shocks, a firm's change in total sales over all industries identifies input scalability, whereas its change in relative sales across industries identifies input mutability. Intuitively, just as relative input cost shifters identify substitutability across inputs in the production of a given output, in my setting relative output profitability shifters (demand shocks) identify mutability across outputs in the use of a given input.

[3]Existing papers, on the other hand, study the effects of specific knowledge inputs in isolation—ranging from R&D (Aw et al., 2011), marketing (Arkolakis, 2010), management (Bloom et al., 2019), to ICT (Fort, 2016; Lashkari et al., 2019)—and focus on the scalability of these inputs in single-industry production rather than their mutability across industries.

[4]A contemporaneous paper by Boehm et al. (2019b) also emphasizes heterogeneity across industries but focuses on the role of *physical* intermediate inputs for linking up a firm's entry decisions across industries.

[5]Most papers estimate joint production by appealing to cost duality concepts (Baumol et al., 1982) for just a few products at a time (e.g. agricultural cooperatives in Pokharel and Featherstone, 2019). Grieco and McDevitt (2016) and Dhyne et al. (2017) assume a Cobb-Douglas production possibility frontier, which presupposes diseconomies of scope. De Loecker et al. (2016) and Orr (2019) estimate productivity in multi-product firms by *a priori* assuming that production is independent across industries.

I conduct inference using simulated method of moments. I estimate production parameters in the model that rationalize the same within-firm, cross-industry covariances of sales growth and demand shocks in the data. Consistent with the reduced-form evidence, I allow knowledge inputs to have different scale and mutability parameters from residual capital inputs. BEA data on input expenditures by industry provide variation needed to differentiate properties of knowledge inputs from residual capital inputs. For example, a more positive response of MRI machine sales to aviation demand shocks suggests that the inputs more intensively used in the two industries (e.g. scientists) are more scalable and mutable compared to other capital inputs.

I estimate that knowledge inputs are relatively more scalable and mutable compared to other capital inputs. These properties lead to heterogeneous cross-industry price elasticities of supply within the firm. Elasticities are more negative among knowledge-intensive industries like aerospace and electromedical apparatus manufacturing, consistent with the reduced-form evidence. Despite its parsimony, the estimated model replicates non-targeted cross-sectional moments such as the firm scope distribution and joint production patterns across the entire US manufacturing sector.

Lastly, Section 4 quantifies the extent to which joint production within the firm matters in the aggregate. I focus on aggregate supply elasticities—the response of the producer price index in one industry to market size shifters in another industry. This cross-industry impact would be zero under independent production but non-zero and ambiguous in sign under joint production. I express this matrix of own- and cross-industry macro elasticities in terms of scalability and mutability parameters. Given the estimates, aggregate economies of scope—a fall in prices in one industry in response to an increase in market size in another—are stronger whenever the two industries share more knowledge inputs.

I find quantitatively large economies of scope in the US manufacturing sector. I calibrate the model to US data in 2007 and estimate the impact of a proportional foreign demand shock in each US industry. I decompose the aggregate response of producer prices into changes that occur as a result of industry scale economies (own-industry elasticities) versus scope economies (cross-industry elasticities). Scope economies contribute to an aggregate elasticity of prices to market size of -0.04, more than 20% of the total response of prices. Estimates of own-industry returns to scale—the remaining 80%—are consistent with recent estimates using industry-level data by Kucheryavyy et al. (2019) and Bartelme et al. (2019). These papers, however, *a priori* assume zero cross-price elasticities of supply and would miss 20 percent of the overall price response.

These aggregate numbers mask significant heterogeneity across industries. Industries that use more knowledge inputs are stronger transmitters and beneficiaries of economies of scope. For example, when demand increases in the electromedical apparatus industry, 60% of the total decline in the manufacturing sector price index manifests in *other* industries. As firms respond by scaling up knowledge inputs, the mutability of knowledge generates capital improvements (price declines) in the other industries of these firms that benefit most from similar types of knowledge. But in the flavoring syrup industry, which relies predominantly on other forms of (less scalable and less mutable) capital, an increase in demand actually leads prices in other industries to *rise* due to firms reallocating scarce capital towards flavoring syrup.

As a proof of concept, I show that joint production changes the consumer price impact of US trade policy relative to existing quantitative models surveyed in Costinot and Rodríguez-Clare (2014). I analyze the impact on the US economy of a 20% unilateral tariff on all imports from China. Import protection raises the prices of foreign goods but also lowers prices of domestic goods through scale and scope economies induced by improved market access for domestic firms. The US manufacturing CPI rises by 0.8% in my model compared to 1.12% absent the response of domestic producer prices (i.e. if marginal costs were constant). I also show that knowledge of cross-price elasticities of supply can be used to improve policy. I identify alternative tariff schedules that achieve the same reduction in US imports from China (41%) but more than halve the adverse CPI impact (to 0.38%) by biasing protection towards knowledge-intensive industries with stronger internal economies of scale and scope.

These results contribute to a prevailing literature in macro and trade where industry linkages are driven by interactions *across* rather than within firms. I provide the requisite microeconomic evidence to distinguish internal joint production from external effects such as agglomeration externalities (Ellison et al., 2010) and innovation spillovers (Bloom et al., 2013). My focus on supply elasticities (how demand shocks move prices *along* a supply curve) is distinct from the literature on production networks (how productivity shocks *shift* the supply curve, in Gabaix, 2011; Acemoglu et al., 2012, 2016; di Giovanni et al., 2018; Baqaee and Farhi, 2019; Liu, 2019; Lim, 2018). Nevertheless, these two mechanisms interact to compound the aggregate returns to scale and scope. I find that embedding joint production within an external input-output production structure à la Caliendo and Parro (2014) more than doubles the cross-industry price impact of demand shocks.

1 Data and Empirical Evidence

This section introduces two main data sources: (i) panel data on the sales and exports, by industry, of all US manufacturing firms, and (ii) industry-level variation on input expenditures. I use this data to provide evidence that production is interdependent across industries within a firm. My identification strategy exploits plausibly exogenous changes to foreign market size as shifters of demand specific to an industry within a firm. I find that a demand shock in one industry of a firm increases its sales in another industry the more that both industries share knowledge-generating inputs.

1.1 Joint Production in US Manufacturing Firms

I construct a firm-industry-level panel dataset containing the universe of US manufacturing firms from 1997 to 2012. I make use of product trailer files in the quinquennial Census of Manufactures, which contain sales across detailed product lines within each establishment of a firm. I aggregate these detailed products to a set $\mathcal{J}$ of 206 industries, a level at which I assume production technologies (use of inputs) might differ.[6] I define the sales of a firm f in industry j in year t, X_{fjt}, as the sum of all external shipments of j over all of the firm's establishments.[7] I construct data on these firms' exports by merging in customs data from the Longitudinal Foreign Trade Transaction Database (LFTTD) at the same level of industry clarity.

Table 1 summarizes the prevalence of *multi-industry* firms in US manufacturing, suggesting that joint production within these firms may have first-order effects on aggregate inter-industry relationships.[8] One-fifth of all US manufacturers operate in two or more industries, accounting for around three-quarters of manufacturing sales, exports, imports, and employment.[9] The second and third rows of the table show that sales within these firms are not too skewed towards their primary (highest grossing) industry. The primary industry of each firm accounts for only roughly two-thirds of the firm's total sales. The remaining industries (just a single one for the median multi-industry firm) are classified under 'secondary' and account for one quarter of total gross output.

The median multi-industry firm is active in only two industries. Interestingly, these two industries also fall wide enough to be two different sectors (defined as 3-digit NAICS

[6]This is roughly consistent with 5-digit NAICS codes. It is the most disaggregated level at which input-output flows (across industries) are available in BEA data over time. More details on the match and construction of this concordance can be found in the Data Appendix.

[7]I define external manufacturing sales as the total shipments less inter-plant shipments (from one plant to another plant within the same firm). This classification in the data ignores the plant-dimension and the paper does not distinguish plant-level from firm-level economies.

[8]Bernard et al. (2010) provide a more detailed overview using the same data up to 1997. See Data Appendix A.2 for statistics on the non-manufacturing sector.

[9]A classic explanation for the firm boundary is that firms integrate vertically related industries to avoid holdup and contracting inefficiencies. Consistent with Atalay et al. (2014), I find that within-firm shipments of goods are a small fraction of firms' overall sales and thus appear inadequate for explaining firm scope.

Table 1: Statistics on US Multi-Industry Manufacturing Firms

	1997	2002	2007	2012
Share of aggregate outcome				
Manufacturing sales	.75	.75	.75	.76
by primary industry	.46	.49	.51	.52
by secondary industries	.29	.26	.24	.24
External manufacturing sales, X_{fj}	.74	.74	.74	.75
Manufacturing employment	.62	.63	.60	.60
Exports	.84	.80	.81	.76
Imports	.82	.79	.79	.77
Number of firms	.19	.20	.20	.20
Mean and median scope				
Mean number of industries	2.69	2.73	2.63	2.65
Median number of industries	2	2	2	2
Mean number of sectors	1.69	1.74	1.69	1.70
Median number of sectors	2	2	2	2

Notes: Multi-industry firms are firms producing manufacturing products that fall in at least two distinct industry classifications. An industry is defined at a hand-constructed, roughly 5-digit NAICS level, of which there are 206 across the manufacturing sector. A sector refers to a 3-digit NAICS code, of which there are 21 in manufacturing. External manuf. sales is equal to the firm's gross manufacturing sales less its total inter-plant shipments reported.

codes), consistent with the view that production within a firm spans sufficiently distinct production processes. This broad classification of what is an industry sets my empirical investigation apart from other papers that study the product-variety margin of the multi-product firm (Feenstra and Ma, 2007; Arkolakis et al., 2019; Macedoni and Xu, 2019).

1.2 Knowledge Inputs in Production

I focus on a category of inputs—knowledge-generating inputs—that constitute 15 percent of US GDP but whose role in the production process is less well understood than physical intermediate inputs. I classify knowledge inputs as the following NAICS industry categories (root code in parentheses): professional and technical services (54), management of companies and enterprises (55), the leasing of intangible assets (533), and information (51). These inputs are chosen to reflect their potential contribution towards knowledge capital (or intangible capital) within the firm.[10]

Figure 1 plots expenditures on knowledge inputs as a share of gross output across US manufacturing industries in 1997 (ranked in ascending order on the x-axis). I use the input-output tables and capital flow tables from the Bureau of Economic Analysis to infer input-by-industry expenditure shares. I define expenditure shares of industry j on any input m as:

$$\beta_{jm} \equiv \frac{E_{jm} + I_{jm}}{\sum_m E_{jm} + I_{jm}},$$

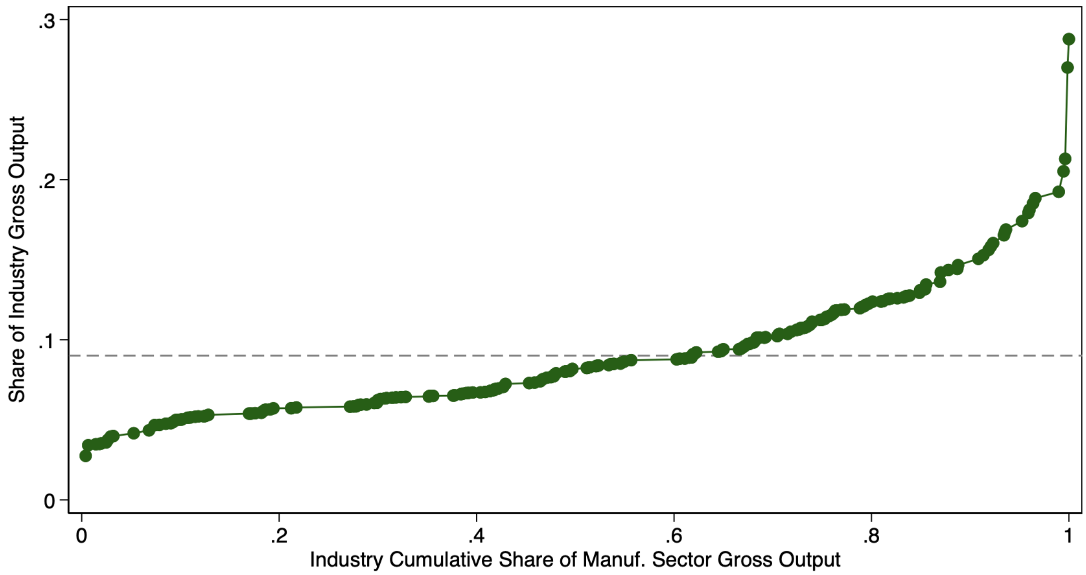
where E_{jm} are the expenses of industry j on input m and I_{jm} are the capitalized investments on input m made by industry j to form new capital.[11]

On average, expenditures by US manufacturing firms on knowledge inputs amount to over 9% of gross output (given by the dashed line). There is substantial variation across industries: the 25th-percentile industry spends 6.4% of gross output on knowledge, while the 75th-percentile industry spends nearly double that—11.5% of gross output—on knowledge. These expenditures are substantial given that labor inputs account for on average only 19% of gross output.

[10] See the Data Appendix Section A.1 and accompanying Table 7 for more detail on the exact input industries. Examples of knowledge inputs include data processing services, scientific R&D, engineering, consulting, architectural, advertising, and legal services.

[11] I account for both capitalized and expensed inputs from the knowledge sector. But because the capital flow table ceases to exist (at least publicly) after 1997 and national accounting rules on the capitalization of intangibles change over time, I am unable to analyze how expenditures change over time. I hold the shares fixed at their 1997 values through the empirical analysis in this section. Expensed expenditures on knowledge in 1997 account for the vast majority of spending, so results are robust to using just expenditure flows from the I/O use table.

Figure 1: Expenditures on Knowledge-generating Inputs in US Manufacturing, 1997



Notes: This figure shows the share of expenditures on knowledge inputs, by manufacturing industry, in increasing order of expenditure share. The x-axis tracks the cumulative share of gross output in manufacturing by the industry and all preceding industries in rank.

1.3 Reduced-form Evidence of Production Interdependence

I use the firm panel dataset and industry-level expenditure shares to test whether production is independent across the industries of a firm. Under the null hypothesis, a profit maximizing multi-industry firm operates independent production processes across industries with separate input and output markets. An increase in demand in a given industry of the firm would increase sales in that same industry but have no effect at all on sales in any of the firm's other industries. On the other hand, if production were jointly determined across industries, an increase in production in one industry of the firm might generate spillovers in other industries.

I carry out this test using the following empirical specification. I examine whether sales growth in a given industry j of a firm ($\Delta \log X_{fjt}$) is affected by a demand shock in any of the firm's other industries ($\Delta \log S_{fjt}^{OTHER}$), conditional on the impact of any own-industry demand shocks ($\Delta \log S_{fjt}$):

$$\Delta \log X_{fjt} = \psi^{OWN} \, \Delta \log S_{fjt} \; + \; \psi^{SPILL} \, \Delta \log S_{fjt}^{OTHER} + \\ Controls_{fj,t-1} + FE_{jt} + \epsilon_{fjt}, \qquad \forall f, j, t = \{2, 3\}, \quad (1)$$

where $t = 1, 2, 3$ refer to the years 1997, 2002, and 2007, and Δ is a first-difference operator between t and $t-1$ (a five-year difference in the data).[12] Industry-year fixed effects sweep away any supply and demand shocks common to all firms in an industry, while $Controls_{fj,t-1}$ account for non-parallel growth trends that depend on initial firm-industry characteristics (such as size or export intensity).

Under the null hypothesis, the coefficient ψ^{SPILL} would be equal to zero whenever $\Delta \log S^{OTHER}_{fjt}$ references a firm's demand shock in *any* other industry $k \neq j$. In principle, this is a test that each of the $J(J-1) = 42230$ bilateral cross-elasticities ψ^{SPILL}_{jk} is zero. I do not have statistical power to separately test each cross-elasticity given my limited sample size. Instead, I test two necessary conditions for the null to hold: (i) whether cross-elasticities are zero on average, and (ii) whether cross-industry shocks among certain types of industries (e.g. those that share more knowledge inputs) have a differential impact from the average.

Demand Shocks

I use the five-year change in imports (excluding from the US) in a foreign market (denoted $\Delta \log IMP^{\sim US}_{nht}$, for a product h in a country n) as a proxy for a shift in market size faced by a US exporter selling in that market. I construct demand shocks, $\Delta \log S_{fjt}$, by interacting these shifts in market size with pre-existing variation in a firm's export shares across markets within an industry:

$$\Delta \log S_{fjt} = s^*_{fj,t-1} \sum_n \sum_{h \in H_j} s_{fnh|fj,t-1} \Delta \log IMP^{\sim US}_{nht}, \tag{2}$$

where $s^*_{fj,t-1}$ is the firm's export intensity in industry j in year $t-1$ and $s_{fnh|fj,t-1}$ is the share of the firm's exports in industry j that go to destination n and HS6 product h.[13]

I use annual customs data on firm exports by destination and product to construct the export intensity and share variables, and the BACI Comtrade dataset on annual global trade flows between countries at the HS6 level to construct the import growth

[12] I drop the time period 2007-2012 in all but summary statistics because (i) the financial crisis generated correlated shocks across countries, industries, as well as firms, jeopardizing variation in the instrument, and (ii) the relevance of variation in input expenditure shares (which I hold fixed to 1997) is diminished.

[13] Existing papers use this shift-share identification strategy to construct demand shocks at the firm-level, using data from countries such as Denmark (Hummels et al., 2014), France (Mayer et al., 2016), and Portugal (Garin and Silverio, 2018). In my paper, the scale and scope of US exporters allows demand shocks to vary across industries *within* the firm. Similar to Aghion et al. (2019), I match census and customs data sources on firm exports to construct demand shocks. There are roughly 5000 HS6 product codes h, and only 206 industries j, with very few instances of a HS6 code h concording to more than one industry j. See Data Appendix Section A.3 for how I deal with carry-along trade and product-industry concordance splits.

variable.[14] My empirical setting benefits from variation induced by the large number of combinations—over 1 million—of markets (nh pairs). Differences in import growth across these markets drive variation in the shift-share measure of a firm's demand shock in a given industry j. The scale and scope of US multi-industry exporters also contributes to variation in the demand shock, even across firms within an industry. The median number of product-destination export markets within an industry of a firm in my sample is 6.2, and the mean is 24.1.

My identification assumption is that firm-industry specific demand shocks are conditionally uncorrelated with (i) unobservable supply-side shocks (that shift the cost function) and (ii) unobservable demand-side shocks (that shift product demand) in the firm's other industries. One threat to identification is that changes in the imports of foreign markets (from non-US sources) could still reflect idiosyncratic supply-side shocks within a large-enough US firm. For example, Indian imports of X-ray scanners from other countries could fall if GE, a major exporter, became more productive at making them (and thus exported more to India). To mitigate contamination from these unobserved supply-side shocks, I construct the demand shock in equation (2) using only variation from the export markets of a firm in which it has a market share below 10%.[15]

Other-Industry Demand Shocks and Knowledge Input Proximity

I construct two different types of weighted averages of a firm's shocks in other industries. First, I use a sales-weighted average that assumes demand shocks have spillover effects proportional to their current size (gross sales) within the firm:

$$\Delta \log S_{fjt}^{OTHER,SYM} \equiv \sum_{k \neq j} \left(\frac{X_{fk,t-1}}{\sum_{k \neq j} X_{fk,t-1}} \right) \Delta \log S_{fkt}.$$

However, the sales-weighted average pre-supposes a common sign for spillovers across industries. In reality, some pairs of industries may be complements in production while other pairs substitutes. I explore an alternative functional form that tests

[14] Changes in import growth $\Delta \log IMP_{nht}^{\sim US}$ could reflect both (i) changes in the level of demand in that market, and (ii) changes in the degree of foreign or home producer competition in that market that affect the price index. Both sources of variation are relevant shifters of a US firm's residual demand, though they would move residual demand in opposite directions. I find that empirically, the first force dominates.

[15] Empirically, my reduced-form results are robust to different threshold choices.

whether spillovers are correlated with the intensity of use of knowledge inputs $\mathcal{M}^{KLG}$:

$$\Delta \log S_{fjt}^{OTHER,KLG} \equiv \sum_{k \neq j} \sum_{m \in \mathcal{M}_f^{KLG}} \beta_{jm} \left(\frac{\beta_{km} X_{fk,t-1}}{\sum_{k \neq j} \beta_{km} X_{fk,t-1}} \right) \Delta \log S_{fkt},$$

where the sales-share weights in the SYM functional form are replaced with a product of two terms. The first term, β_{jm}, is the expenditure share of industry j on input m. The second term (in parentheses) measures the importance of industry k in the firm's overall spending on input m relative to the firm's other industries $k' \neq j$. This multilateral weighting scheme assumes that a demand shock in an industry k should matter more than a shock in k' for affecting sales in j whenever k spends more (compared to k') on the specific knowledge inputs $m \in \mathcal{M}_f^{KLG}$ that are used intensively in j.

Reduced-Form Results

Table 2 presents estimates of spillover coefficients ψ^{SPILL} based on equation (1) using the export demand shocks and various functional forms for $\Delta \log S_{fjt}^{OTHER}$. My sample includes all firms that are multi-industry in each base year $(t-1)$ for which empirical measures of demand shocks can be constructed in at least one industry. Observations are continuing industries of these firms over each five-year period.[16] Appendix Table 8 provides summary statistics on regression variables and other attributes of firms in the sample. The final regression sample of roughly 5000 multi-industry continuing firms per year accounts for over half of all US manufacturing gross output.

I test whether the coefficients $\psi^{SPILL,SYM}$ and $\psi^{SPILL,KLG}$ are zero, which are necessary conditions for the null hypothesis of independent production across industries. I also test whether within-firm spillovers across industries vary with the intensity of use of knowledge inputs—if $\psi^{SPILL,KLG}$ is different from $\psi^{SPILL,SYM}$.

First, column (1) reveals that the demand shocks are empirically relevant. A demand shock in one industry of the firm increases the firm's sales in that same industry relative to other firms.[17] The next two columns, (2) and (3), show that spillovers from demand

[16] For example, suppose firm f produces in industries A and B in 1997 but only produces in A in 2002. As long as the firm received a demand shock in either industry A or B in 1997, I include the firm in the sample (where it takes up a single observation, f =f, j =A). However, if in 2002 the firm switches to producing industries C and D, there is no intensive margin overlap and this firm does not appear in my sample.

[17] Appendix Table 9 shows that this 'first-stage,' own-shock relevance is not driven by the export intensity variable $s^*_{fj,t-1}$ picking up pre-trends in growth rates (i.e. if more export-intensive industries of the firm grow faster). Estimates are robust to including as controls a full set of industry-year dummy variables interacted with the export intensity variable. This is recommended by Borusyak et al. (2018) for specifications that work with 'incomplete' shift-share research designs.

Table 2: Cross-Industry Spillovers within the Firm

Sales growth, $\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)	(5)	(6)
Own-industry shock	0.45***				0.46***	0.37*
$\Delta \log S_{fjt}$	(0.10)				(0.09)	(0.19)
Cross-industry shocks						
(i) Knowledge-based spillovers			0.81	7.51***	8.00***	13.31***
$\Delta \log S_{fjt}^{OTHER,KLG}$			(1.01)	(2.22)	(2.25)	(3.58)
(ii) Symmetric spillovers		-0.03		-0.74***	-0.83***	-1.67***
$\Delta \log S_{fjt}^{OTHER,SYM}$		(0.11)		(0.24)	(0.24)	(0.52)
Industry-year-FE	✓	✓	✓	✓	✓	✓
Firm-year-FE						✓
Observations	21,500	21,500	21,500	21,500	21,500	17,500
R^2	0.06	0.06	0.06	0.06	0.06	0.39

Notes: This table displays responses of firm-industry sales to demand shocks across the firm's range of industries, in 5-year differences over the period 1997-2007. Standard errors are clustered at the firm level. Number of observations are rounded for disclosure avoidance. Observations are at the firm-industry-year level, for continuing observations of a sample of multi-industry firms with at least one industry directly exporting. Results are unweighted but robust to weighting by the inverse within-firm share of sales of industry j. Results are robust to deflating outcomes and also shocks.

shocks in other industries appear to be a wash when evaluated separately. The average cross-elasticity, $\psi^{SPILL,SYM}$, is statistically insignificant from zero. This could occur for one of two reasons: either (i) production is independent across industries, or (ii) some pairs of industries are complements in production while others are substitutes, so that the net effect comes out to zero.

The remaining columns, starting with column (4), indicate that spillovers are stronger and more positive among industry pairs that share more knowledge inputs. Controlling for this positive effect concentrated around knowledge-intensive industries, the remaining average spillover is negative. These coefficients are consistently estimated and statistically significant across a battery of controls in spite of substantial collinearity around the two measures of cross-industry shocks.[18]

Column (5), the preferred specification, accounts jointly for own as well as cross-industry demand shocks. Reassuringly, controlling for own-industry demand shocks does nothing to change the magnitude or significance of cross-industry spillovers. Column (6) shows that results remain robust to controlling for firm-year fixed effects, which soak up

[18] This collinearity—at 0.9—is consistent with regression columns (2) and (3) being independently insignificant due to omitted variables bias.

unobserved supply and demand shocks that jointly affect all industries of the firm.[19]

While the quantitative magnitudes of these reduced-form elasticities are best assessed through the lens of an economic model (provided in Section 2), the following back-of-the-envelope calculation reveals quantitatively large spillovers. Consider a firm that operates an industry j that has a knowledge input proximity with the the firm's other industries, $k \neq j$, of 0.09, the mean value in the sample. If this firm receives a common demand shock in all of its industries equal to one standard deviation in the sample (8.2%), the firm's sales in industry j would rise by 2.7%. This total impact comprises an own-industry response (3.8%) and a net negative cross-industry response (-1.1%).

However, the net cross-industry spillover turns positive for industry pairs that share more knowledge inputs. For another firm receiving the same shocks but whose industry j is more proximate (by one standard deviation above the mean) in its use of knowledge inputs with its other industries $k \neq j$, the cross-industry response rises to a positive 1%. The total impact of demand shock on sales in industry j is now 4.8% (= 3.8% + 1%), with cross-industry spillovers accounting for more than one-fifth of this total response.

Robustness to Controlling for Covariates

The estimates of spillovers in Table 2 are robust to controlling for an exhaustive set of firm-industry level covariates. These covariates—including pre-period firm-industry size, export intensity, export status, and firm-wide attributes—account for the possibility that industry segments of firms might be on different growth trends. Moreover, I construct *other-industry averages* of these covariates by interacting them with the same weights used to average demand shocks in other industries of the firm. For any firm-industry level covariate $Y_{fk,t-1}$, I construct:

$$Control^{CROSS,KLG}_{fjt}(Y) \equiv \sum_{k \neq j} \sum_{m \in \mathcal{M}^{KLG}_f} \beta_{jm} \left(\frac{\beta_{km} X_{fk,t-1}}{\sum_{k \neq j} \beta_{km} X_{fk,t-1}} \right) Y_{fk,t-1},$$

and likewise for $Control^{CROSS,SYM}_{fj}(Y)$ using sales shares over $k \neq j$ as weights. These variables account for the possibility that sales growth in any industry j of the firm is correlated with either pre-existing covariates (such as size) in *other* industries $k \neq j$ or input proximity weights.

In column (1) of Appendix Table 10, I saturate the baseline regression specification

[19]In undisclosed results, I also find that the specification is robust to including firm-industry fixed effects, which further limits identifying variation to *differences* in growth rates and demand shocks between the periods 1997-2002 and 2002-2007.

with an exhaustive list of pre-period covariates: firm-wide log sales, export intensity, firm-industry-level log sales, export intensity, export status, as well as the firm-industry covariates passed through the spillover functions $Control_{fjt}^{CROSS,KLG}$ and $Control_{fjt}^{CROSS,SYM}$. Reassuringly, controlling for these covariates affects neither the significance nor magnitude of the estimated spillover coefficients, $\psi^{CROSS,SYM}$ and $\psi^{CROSS,KLG}$.

Spillovers from other types of Input Linkages

Are spillovers correlated with industries' intensity of use of other categories of inputs, besides knowledge? I extend the multilateral input proximity measure to pick up on the sharing of other inputs among shocked and affected industries. I construct these alternative other-industry demand shocks as

$$\Delta \log S_{fjt}^{OTHER,BLK} \equiv \sum_{k \neq j} \sum_{m \in \mathcal{M}_f^{BLK}} \beta_{jm} \left(\frac{\beta_{km} X_{fk,t-1}}{\sum_{k \neq j} \beta_{km} X_{fk,t-1}} \right) \Delta \log S_{fkt},$$

which simply replaces the set of knowledge inputs $\mathcal{M}_f^{KLG}$ used to construct $\Delta \log S_{fjt}^{OTHER,KLG}$ with another subset of inputs $\mathcal{M}_f^{BLK}$.

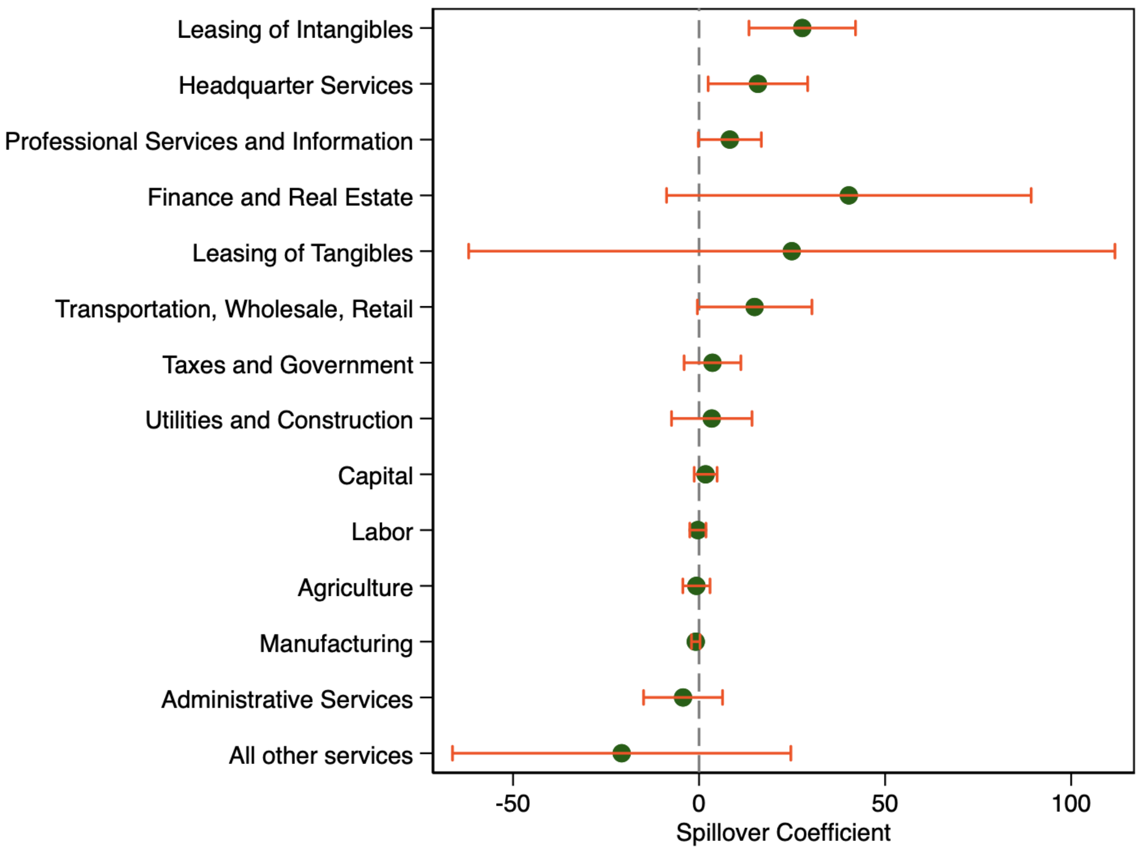
I reproduce the specification in column (5) of Table 2 but replace $\Delta \log S_{fjt}^{OTHER,KLG}$ with $\Delta \log S_{fjt}^{OTHER,BLK}$, one BLK category at a time, across the universe of BEA input categories.[20] Spillover coefficients, $\psi^{SPILL,BLK}$, are displayed in Figure 2 and the corresponding regression table can be found in Appendix Table 11. The first three rows of Figure 2 break down my classification of knowledge inputs into three sub-categories: headquarter services, professional services and information, and leasing of intangibles. I find that each sub-category is positive and statistically significant.

This comparison suggests that knowledge inputs have plausibly distinct characteristics from other inputs. Spillovers are uncorrelated with the intensity of use of any other category of inputs, with the exception of "transportation, wholesale, and retail" (where positive spillovers are consistent with economies of scale in distribution). In particular, the coefficients relating to input categories such as labor, agriculture, manufacturing, and capital (computed as gross operating surplus) are all precisely estimated at zero, suggesting industries that predominantly use these inputs have plausibly independent production processes.[21]

[20] To enable a fair comparison, I include the same controls as the original column (5)—the own-industry shock and the sales-weighted spillover function, $\Delta \log S_{fjt}^{CROSS,SYM}$.

[21] Lack of statistical power prevents joint estimates of spillover elasticities for each input block in the same regression. Whenever three of more "other-shock" averages are included, collinearity renders all estimated

Figure 2: Cross-Industry Spillovers by Category of Input



Notes: This figure displays estimates of spillover coefficients $\psi^{CROSS,BLK}$ in alternative versions of regression specification (5) of Table 2, where BLK references the input category across the rows of the Figure. Point estimates are in green and 95% confidence intervals in orange. See Appendix Table 11 for the corresponding regression table and for more details on the industry codes of these input categories.

1.4 Interpretation and Discussion of Results

Spillovers are stronger and more positive the more that industries share knowledge inputs. While the average spillover coefficient, $\psi^{SPILL,SYM}$, can in principle be explained by non-production-related mechanisms such as demand cannibalization or financial frictions, I find that these spillovers are on average a wash. The coefficient $\psi^{SPILL,SYM}$ is not independently significant. In this subsection I discuss plausible mechanisms that explain these findings as well as threats to identification.

Mechanism: Scalability of Shared Knowledge Inputs

The positive correlation of spillovers with knowledge input use is suggestive of a supply-side mechanism: when firms scale up knowledge inputs in response to a demand shock in one industry, the inputs might also improve knowledge capital in *other* industries within the firm.

I use firm-level input expenditure data to provide evidence on a necessary condition behind this transmission mechanism. I show that firm-wide expenditures on a particular category of observable knowledge inputs—purchased professional services—rise in response to firm-wide demand shocks, consistent with the hypothesis that these inputs are scalable.[22] Table 3 estimates the elasticities of a set of firm-level outcome variables, Y_{ft}, to firm-level demand shocks:

$$\Delta \log Y_{ft} = \tilde{\nu} \sum_{k} \eta_{fk,t-1} \Delta \log S_{fkt} + \epsilon_{ft}, \tag{3}$$

where $\eta_{fk,t-1}$ are weights that measure the relative propensity of demand shocks in industry k to shift firm-wide outcomes, depending on the outcome variable.[23]

I find that purchased professional services (in column 1) rise in response to firm-wide demand shocks, with an elasticity of 0.65.[24] This elasticity reflects the joint effect of input scalability and the responsiveness of marginal revenue to the empirically constructed

ψ^{CROSS} coefficients insignificant except $\psi^{CROSS,KLG}$, which always retains significance. See, for example, specification (4) of Appendix Table 10, which controls jointly for sales, physical-input, and knowledge-input weighted other-industry shocks.

[22]Such firm-level data is available for only a subset of firms. Purchased professional services comprise of purchased software and data processing, management, and advertising that are outsourced by the firm and represent a plausible subset of knowledge inputs in BEA tables.

[23]When the outcome variable is professional services, weights equal industry relative expenditures on knowledge inputs (imputed from BEA I/O table coefficients). When the outcome variable is sales, weights equal industry relative sales. See Data Appendix A.4.5 for the precise definitions.

[24]These results are consistent with papers that document a positive relationships between R&D and market size (Aghion et al., 2019), and IT and market size (Lashkari et al., 2019) using French data.

Table 3: Firm-level Margins of Response to Demand Shocks

	(1) Purchased Prof. Services	(2) Capex	(3) Payroll	(4) Sales
Outcome-relevant demand shock	0.65***	0.47	0.25**	0.37***
$\sum_k \eta_{fk,t-1} \Delta \log S_{fkt}$	(0.22)	(0.37)	(0.11)	(0.15)
Year-FE	✓	✓	✓	✓
Observations	3,900	3,900	3,900	3,900
R^2	0.02	0.04	0.01	0.05

Notes: This table displays responses of firm-level outcomes to demand shocks averaged across the firm's industries, in 5-year differences over the period 1997-2007. Standard errors are clustered at the firm level. Number of observations are rounded for disclosure avoidance. Observations are at the firm-year level, for continuing firms with at least one industry directly exporting and reporting non-zero purchased professional services.

demand shifter. I revisit and leverage this reduced-form coefficient when structurally estimating the model in Section 3.[25] In comparison, capital expenditures and payroll (columns 2 and 3) all respond with a lower elasticity than purchased professional services, consistent with the hypothesis that knowledge-generating inputs are more scalable than these other inputs. Finally, column (4) estimates the firm-level elasticity of sales to demand shocks at 0.37, a value in between the elasticity of various input expenditures in the prior columns. This provides reassurance that not all of the observed response of firm sales is due to an adjustment in markups.

Unfortunately, there is no information on how the input expenditures of a firm (e.g. data processing services) are apportioned across industries.[26] The potential for any given input to jointly contribute to output across multiple industries is not separately testable; it is implicit in the reduced-form evidence. The model that I subsequently develop inherits this general mapping from inputs to outputs. Estimation does not rely on firm specific observations of expenditure use by industry. Variation in input-by-industry expenditure data contained in aggregate BEA tables is sufficient, and even advantageous. Since proximity weights, β_{jm}, are constructed using industry-level data, the reduced-form estimates are less likely to be correlated with firm-specific unobservables (such as differences in productivity or access to certain inputs), a potential source of endogeneity bias.

[25] This consistency with theory is the primary motivation for using relative input-expenditures $\eta_{fk,t-1}$ instead of relative industry sales as weights. However, results are robust to using relative industry sales as weights across all four outcome variables in Table 3.

[26] Even if such attribution existed, a firm could very well book the initial use of inputs in one industry while reaping benefits in other industries. This data problem is also prevalent in other papers that estimate multi-output production functions (De Loecker et al., 2016; Orr, 2019). The solution taken in these papers is to assume separable production functions so that input use can be apportioned.

Other Interpretations and Threats to Identification

For the spillovers in Table 2 to be interpreted as evidence of joint production, demand shocks in the firm's *other* industries $k \neq j$ would have to be conditionally uncorrelated with unobserved shifters of sales in a given industry j of the firm.[27] Unobserved shifters of sales could take the form of either demand and supply shocks and induce omitted variables bias.

My empirical strategy is resilient to omitted variables bias due to both demand and supply unobservables. First, the lack of significance when either of the cross-industry shock variables are evaluated separately (in columns (2)-(3) of Table 2) rule out a simple correlation structure between unobserved shifters and export demand shocks.

Nevertheless, on the demand side, import growth in foreign markets might just happen to be more positively correlated across industries that share more knowledge inputs. In Appendix Section A.4.6, I directly test and reject this hypothesis in the trade data. A related concern is that industries that share more knowledge inputs are also disproportionately complementary in demand. If true, a demand shock in only one of these industries is enough to trigger an increase in sales in both. To mitigate this concern, I show that results continue to be significant when I purge from the sales of industry j all exports to destination countries where the demand shocks for *other* industries k originated.[28]

On the supply-side, my preferred interpretation is that firm-specific supply-side innovations are the precise result of these cross-industry demand shocks, and therefore a part of the estimated spillover response. There is, however, a less organic interpretation: firms that anticipate positive supply-side shocks in a pair of industries j, k select into exporting in those industries (and, in particular, to faster-growing markets with higher demand shocks). For this interpretation to also explain the null coefficients in columns (2) and (3) of Table 2, firms would have to be systematically better at anticipating these effects in knowledge-input-intensive industries. Suppose, for the sake of argument, that this were true. Sales growth would become a function of pre-existing joint production and exporting patterns (driven by supply shocks observable to the firm but unobservable to the econometrician). But I show that this hypothesis does not survive the following

[27]However, a demand shock in industry k can be arbitrarily correlated with unobserved supply and demand shocks exclusive to that same industry—this would merely change the interpretation of the own-industry elasticity, ψ^{OWN}. Demand shocks also do not need to be unanticipated. Regression coefficients pick up precisely the endogenous supply side response in industry j to demand shifters in industry k. The possibility that a particular shock is anticipatable t years ahead of time simply changes the interpretation of the relevant time horizon for a supply-side response to materialize.

[28]Results are also robust to controlling for *latent* demand shocks—a measure of demand for industry j of the firm not from where it is *currently* exporting the goods (which is $\Delta \log S_{fjt}$) but from the other export destinations to which it currently sells goods in other industries.

placebo exercise. I re-assign firm-industry exporters in each industry k different export demand shocks drawn from the empirical distribution of shocks in that industry. Keeping all remaining firm variables (e.g. firm-industry production weights, and other controls) the same in these placebo regressions, I do not find a statistically significant number of positive spillover coefficients.[29]

Finally, in Appendix Section A.4.7, I test and reject other mechanisms that could explain the spillovers, such as vertical (both upstream and downstream) intra-firm relationships between k and j.

2 Quantitative Framework of Joint Production

This section develops a quantitative model of joint production to interpret and quantify the reduced-form evidence. I propose a general mapping from inputs to outputs where two properties of inputs lead to production interdependence: scalability and mutability. Within the firm, the more that a pair of industries share scalable, mutable inputs in production, the higher is the cross-industry impact of demand shocks on sales. I embed this flexible supply-side within an otherwise standard setting featuring monopolistic competition and CES demand, generalizing workhorse models of heterogeneous firms (Melitz, 2003) by allowing for variable cross-industry price elasticities of supply within the firm.

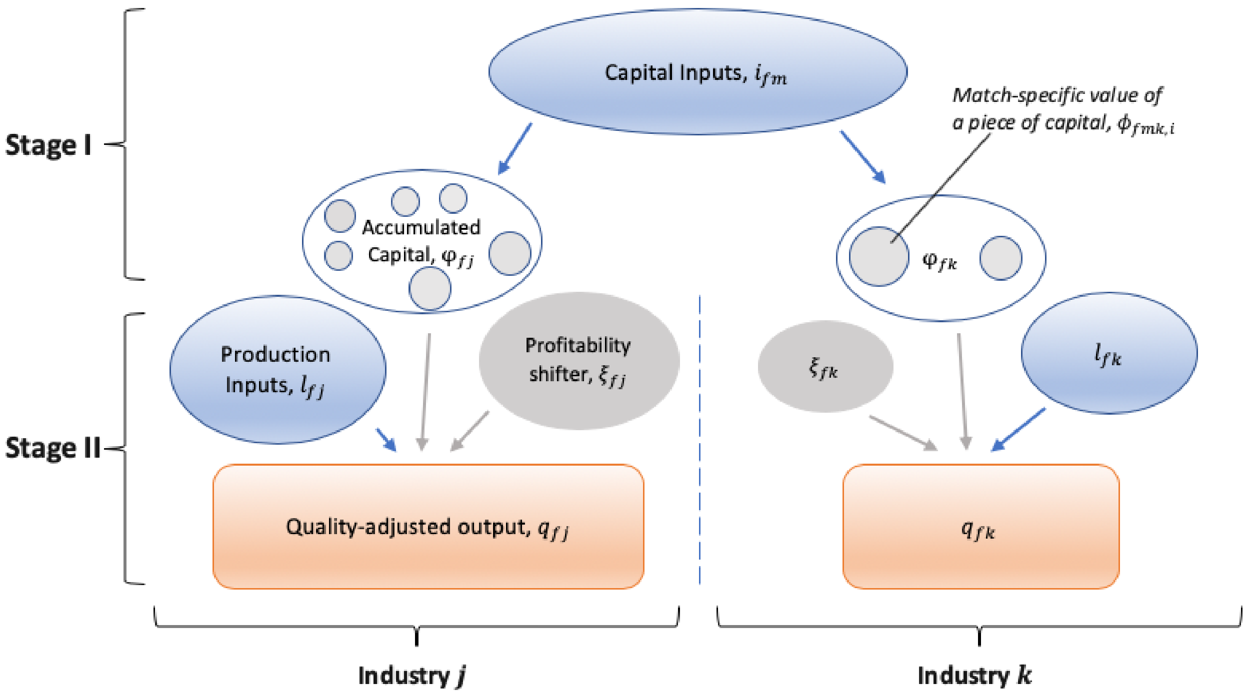
2.1 Market Structure and Production Technology

A continuum of firms compete in a set of industries $\mathcal{J}$ under monopolistic competition facing CES demand with elasticity σ_j. I abstract from demand complementarities across industries, strategic interactions across firms, and variable markups so that the only source of interdependence in a firm's profit maximization problem comes from properties of joint production.

Figure 3 provides a visual illustration of the joint production technology—the process by which a firm's use of inputs generate outputs across industries. The transformation of inputs to outputs occurs over two stages. In stage I, the firm uses inputs to accumulate composite levels of capital in each industry. In stage II, the firm spends on variable production inputs, which, together with capital, generate output in each industry.

[29] These results are undisclosed but available upon request. This is also a placebo test to make sure that pre-existing variation in bilateral industry characteristics such as the extent of knowledge-input sharing do not drive spillovers.

Figure 3: Illustration of Joint Production Technology within the Firm



Notes: This figure provides a visual illustration of the firm's two-stage joint production process for a given firm. There are two types of inputs (displayed in blue): stage I inputs are called capital inputs, while stage II inputs are called production inputs. Capital inputs are used to generate a discrete set of various types of capital (displayed by the small grey circles) that together make up the firm's accumulated capital (displayed in white). Quality-adjusted outputs are denoted in orange. Profitability shifters (displayed in grey) are exogenous and observable by the firm in both stages. The technology extends easily from the displayed case of two industries j, k to accommodate additional industries $k', k'', \ldots$ The only difference would be an increased range of options for the stage I allocation of capital.

Assumption 1 (Industry Production Function) *In each industry j, quality-adjusted quantities q_{fj} is a Cobb-Douglas function over three terms: (i) a composite production input, l_{fj}, (ii) accumulated capital, φ_{fj}, and (iii) an exogenous profitability shifter, $\tilde{\xi}_{fj}$:*

$$q_{fj} = l_{fj}^{\gamma_j} \, \varphi_{fj} \, \tilde{\xi}_{fj} \tag{4}$$

where $\gamma_j \in [0, \frac{\sigma_j}{\sigma_j - 1})$ is the elasticity of output with respect to the production input composite, l_{fj}.

I segment the universe of inputs into two categories and assume that firms are able to purchase each type of input at constant unit prices. The first category consists of *production* inputs, such as labor, materials, and energy, that together make up a homothetic constant-returns composite production input denoted by l_{fj}.

The second category, *capital* inputs, includes knowledge workers like scientists as well as physical equipment.[30] The firm uses these inputs to accumulate capital (φ_{fj}) in each of its industries. Accumulated capital reflects the composite value of attributes such as robotics, assembly-line productivity, customer lists, warehousing capabilities, product design, and brand capital. 'Capital' in the model is thus a broad catch-all that includes both intangible knowledge assets and specialized capital equipment that have been tailored to the specific production process of the firm.

Lastly, the production function includes shifters (both demand and supply), $\tilde{\xi}_{fj}$, that stand in for the firm's exogenous levels of brand appeal, capital or technology in each industry. The firm makes input purchase, capital allocation, and production decisions conditional on knowledge of $\tilde{\xi}_{fj}$.

The firm's problem is to spend on inputs given constant unit prices to maximize total profits (revenues net of costs) across all industries. Relative to the case of a single-industry production function, the multi-industry firm faces a potential tradeoff between capital allocation in one industry versus another. However, in line with the empirical evidence, production inputs in stage II are still assumed to be separable across industries. Given these assumptions, it is easy to solve for the firm's decisions in reverse order.

2.2 Stage II: Production and Competition

In this last stage, the firm takes its accumulated levels of capital (φ_{fj}) and exogenous shifters ($\tilde{\xi}_{fj}$) as given and thus operates an independent production function in each industry. The firm's problem boils down to maximizing gross profits (π_{fj}, revenues less production input expenditures) under monopolistic competition with CES demand and a non-unitary output elasticity (γ_j) with respect to the composite production input.

The solution to this problem yields gross profits and sales, X_{fj}, as a function of an industry-wide residual profit index, B_j, the exogenous profitability shifter $\tilde{\xi}_{fj}$, and accumulated capital, φ_{fj}:

$$\pi_{fj} = (1 - \varsigma_j)X_{fj} = B_j \xi_{fj} \varphi_{fj}^{\frac{\sigma_j - 1}{\sigma_j(1-\varsigma_j)}}, \tag{5}$$

where $\varsigma_j \equiv \gamma_j \frac{\sigma_j - 1}{\sigma_j}$ is a constant share of industry sales expensed on production inputs, $\xi_{fj} \equiv \tilde{\xi}_{fj}^{\frac{\sigma_j - 1}{\sigma_j(1-\varsigma_j)}}$ is a convenient re-normalization of the exogenous shifter. Lastly, industry-

[30] 'Capital' in the model is purely a semantic label, chosen to acknowledge the traditional assumption that 'capital' levels are determined before variable (production) inputs. There are no ex-ante restrictions on what these inputs actually are in the data. Some of these inputs (such as R&D workers) might fall under the 'knowledge' classification adopted in the reduced-form, while some others (laundry services) might not.

wide residual profits B_j depends on the cost of a bundle of production inputs c_j, the CES price index in the output industry, P_j, and total expenditures in the market, Y_j:

$$B_j = (1 - \varsigma_j) \left(\frac{c_j}{\varsigma_j} \right)^{\frac{\varsigma_j}{\varsigma_j - 1}} \left(P_j^{\sigma_j - 1} Y_j \right)^{\frac{1}{\sigma_j (1 - \varsigma_j)}} . \tag{6}$$

2.3 Stage I: Accumulation of Capital

Whereas physical production in stage II is separable across industries, capital accumulation in stage I is plausibly interdependent across industries. I model this process to reflect two realistic characteristics of capital: discreteness and mutability.

First, capital accumulation is an inherently discrete process. A scientist generates a new invention only once in a while, and the development of new land occurs sporadically. I assume that capital inputs ι_{fm} increase the Poisson arrival rate of new capital:

$$A_{fm} \sim \text{Poisson} \left(Z \left(\frac{\rho_m}{\rho_m - 1} \iota_{fm} \right)^{\frac{\rho_m - 1}{\rho_m}} \right), \qquad \forall m \in \mathcal{M},$$

where $\mathcal{M}$ denotes the set of capital inputs, Z is a technology coefficient, and $\rho_m \in (1, \infty)$ parametrizes scalability. The higher is ρ_m, the more elastic is the arrival rate to increased capital spending.

The firm chooses the best industry in which to allocate each arrival of new capital. Mutability, the second property, reflects variation in the match-specific value of capital across industries. For example, an engineer hired at the onset to improve battery longevity in vacuum cleaners may ultimately generate ideas that are *more valuable* for increasing battery efficiency in electric vehicles. I model the value of a given piece of new capital (denoted i) in a given industry j as an independent Fréchet random variable:

$$Pr(\phi_{fjm,i} \leq x) = e^{-x^{-\theta_m}}, \qquad \forall j \in \mathcal{J}, \ \forall m \in \mathcal{M},$$

where the shape parameter $\theta_m \in (1, \infty)$ parametrizes *immutability*. The higher is θ_m, the lower is the variance of the distribution, and the *less* mutable is capital across industries.[31]

Throughout stage I, a total of A_{fm} pieces of new capital of each type m arrive within the firm. I assume that capital across and within input types $m \in \mathcal{M}$ are additively separable in their contribution to the composite index of accumulated capital.

[31]In the limit as $\theta_m \to \infty$, the distribution collapses to a degenerate point and the value of new capital becomes identical across industries, and fully predictable ex-ante.

Assumption 2 (Accumulation of Capital) *Firms deploy each piece of new capital i of type m to a single industry j knowing its match-specific values across industries $\{\phi_{fjm,i}\}_{j\in\mathcal{J}}$. The cumulative effect on the composite indices of industry-specific capital $\{\varphi_{fj}\}_{j\in\mathcal{J}}$ is additive:*

$$\varphi_{fj}^{\frac{\sigma_j-1}{\sigma_j(1-\varsigma_j)}} = \sum_{m\in\mathcal{M}}\sum_{i=1}^{A_{fm}} \tilde{\alpha}_{mj}\phi_{fjm,i}\mathbf{1}_{fjm,i}, \qquad \forall j \in \mathcal{J},$$

where $\tilde{\alpha}_{mj}$ is a technology parameter governing the average productivity of type-m capital when applied to industry j, and $\mathbf{1}_{fjm,i}$ is the firm's deployment decision variable: equal to 1 if opportunity i from input type m is deployed to industry j, and 0 otherwise.

The technology parameters $\tilde{\alpha}_{mj}$ allow capital in certain industries j to benefit more from the contributions of certain inputs m. Differences in these parameter values generate differences in technological similarity across industries. To take an extreme example, an input m that only produces useful capital in a particular industry j would have all $\tilde{\alpha}_{mk} = 0$ for $k \neq j$, and there would be no capital accumulation tradeoffs across industries from the firm's use of that input.

Crucial for tractability, this functional form assumption makes each new piece of capital additively separable in profits (given by equation 5). The optimal industry in which to deploy any piece of new capital becomes memoryless—independent of past or future decisions. The probability that any new capital of type m is deployed in industry j is given by:

$$\mu_{fmj} \equiv \frac{\delta_{fmj}^{\theta_m}}{\sum_{k\in\mathcal{J}}\delta_{fmk}^{\theta_m}}, \qquad \delta_{fmj} \equiv \xi_{fj}\alpha_{mj}B_j Z, \tag{7}$$

where δ_{fmj} is an exogenous index of residual profitability of input m in industry j in firm f and increasing in the technology parameter (renormalized as $\alpha_{mj} \equiv \tilde{\alpha}_{mj}\Gamma(1-1/\theta_m)$), the firm's productivity shifter in the industry ξ_{fj}, and industry residual demand B_j.

When inputs are more mutable (θ is lower), capital is distributed more evenly across industries. Even industries with inherently low residual profitability could become the optimal target for new capital deployment when match-specific variation in $\phi_{fjm,i}$ is high. On the other hand, when capital inputs are immutable, exceptionally positive shocks in $\phi_{fjm,i}$ become much rarer. New capital becomes deployed repeatedly towards the industry of the firm with the highest residual profitability, δ_{fmj}.

Lastly, discreteness in the capital accumulation process explains the firm's extensive margin without the need for literal fixed costs.[32] Because capital is necessary for produc-

[32] The use of discrete stochastic processes to explain 'zeros' (the absence of firm entry) is inspired by

tion, a firm is active in industry j if and only if it decides to accumulate any capital in that industry. All else equal, the more mutable are capital inputs, the more likely is the firm to deploy new capital over *different* industries, and the wider is the scope of the firm.

2.4 Solution of the Firm

These properties of joint production convexify the firm's profit maximization problem over all industries (which industries to enter, and how much of output in each industry to produce). The first piece of capital allocated to an industry represents industry entry, and any subsequent capital allocation improves capital on the intensive margin. Because of additive separability, a firm is as likely to assign a first piece of capital to an industry as it is a second, conditional on those opportunities arriving.

The firm decides its expenditures on each type of capital input given the degrees of scalability (ρ_m) and mutability (θ_m) behind the capital accumulation process. The solution simply equates the expected marginal revenue of each input with its marginal cost (a constant). Decreasing returns to scale in the ability of capital inputs to increase the arrival rate of new capital ($\frac{\rho_m - 1}{\rho_m} < 1$) guarantees an interior solution.

But how valuable is each piece of new capital? The firm internalizes its ex-post ability to choose the most profitable application among all industries j *after* observing match-specific shocks $\phi_{fjm,i}$. The greater is the variance in the distribution of match-specific shocks, the greater is the expected value of the maximum. Mutability thus generates a form of ex-ante option value for the firm:

Lemma 1 (Option Value from Mutability and Scope) *Given Assumptions 1 and 2, the expected contribution to gross profits of any idea generated by specialized input m is given by Δ_{fm}/Z, where Δ_{fm} is a power sum of residual profitability of input m over industries j :*

$$\Delta_{fm} \equiv \left(\sum_{j \in \mathcal{J}} \delta_{fmj}^{\theta_m} \right)^{\frac{1}{\theta_m}}, \qquad \delta_{fmj} \equiv \xi_{fj} \alpha_{mj} B_j Z, \qquad \forall j \in \mathcal{J}, \quad \forall m \in \mathcal{M}$$

The expected scope of the firm increases with mutability (decreases with θ), and the value of any given idea is increases with mutability and scope.

Klette and Kortum (2004), Eaton et al. (2013), and Armenter and Koren (2014), and present theoretical and computational advantages over models with literal fixed costs. Fixed costs generate non-convexities from the point of view of not just firms but also the aggregate economy. Recent work by Jia (2008), Antràs et al. (2017), and Arkolakis and Eckert (2017) provide algorithms that reduce the computational burden of fixed-cost models but operate under a partial equilibrium framework where residual profits are fixed. Instead, in my stochastic setting each individual firm's profit maximization problem is convex, which reduces dimensionality and guarantees a unique solution for residual profits $\{B_j\}_j$ in multi-industry equilibrium.

There are two illustrative limit cases. As $\theta_m \to 1$, inputs become so mutable that the ex-ante expected value of the best industry application is equal to the simple sum of residual profitability across *all* industries, $\sum_{j\in\mathcal{J}} \delta_{fmj}$. New capital is valued as if it is *non-rival*—contributing jointly and fully in each industry of the firm. On the other hand, as $\theta_m \to \infty$, inputs are completely immutable. The ex-ante expected value of the best industry simply equals $\max_{j\in\mathcal{J}}\{\delta_{fmj}\}$, and option value disappears. The firm makes input purchases with full foresight of which industry the inputs will be used towards building capital.

The firm's expected sales and profits are expressed in closed-form in Lemma 2.

Lemma 2 (The Firm's Solution) *Given Assumptions 1 and 2, the expected gross profits of any firm f in any industry $j \in \mathcal{J}$ is a constant fraction $(1-\varsigma_j)$ of expected sales:*

$$\mathbb{E}[\pi_{fj}] = (1-\varsigma_j)\ \mathbb{E}[X_{fj}] = \sum_m \delta_{fmj}^{\theta_m} \Delta_{fm}^{\rho_m - \theta_m} w^{1-\rho_m}, \tag{8}$$

and the probability that a firm enters industry j, denoted $\chi_{fj} = 1$, is given by

$$Pr(\chi_{fj} = 1) = 1 - \exp\left(-Z \sum_m \delta_{fmj}^{\theta_m} \Delta_{fm}^{\rho_m - 1 - \theta_m} w^{1-\rho_m}\right). \tag{9}$$

Expected net profits of the firm are given by expected stage II gross profits over all industries less stage I expenditures on capital inputs:

$$\mathbb{E}[\Pi_f] = \sum_j \mathbb{E}[\pi_{fj}] - \sum_m w\iota_{fm} = \sum_m \frac{1}{\rho_m} \Delta_{fm}^{\rho_m} w^{1-\rho_m}, \tag{10}$$

where w denotes the constant marginal cost of all capital inputs.

2.5 Industry Spillovers within the Firm

Equation (8) demonstrates how scalability and mutability forces $(\rho_m - \theta_m)$ govern the interdependence of production across industries. A demand shock that increases profitability in industry k (an increase in $B_k \xi_{fk}$) affects the firm's accumulated capital in another industry j through two channels: a substitution effect and a scale effect. First, holding capital spending constant, the firm is more likely to deploy capital in industry k rather than j the more higher is θ_m.[33] This reduces the firm's accumulated capital in j. However, the firm

[33]The use of this Fréchet shape parameter to model choice shares was introduced to trade by Eaton and Kortum (2002). However, rather than focusing on θ as a choice-share elasticity, my paper exploits the role

would also increase its spending on capital inputs, raising the arrival rate of capital the higher is ρ_m. This increases accumulated capital in j holding capital allocation constant. The net effect boils down to a trade-off between substitution (decreasing with mutability) and scale, averaged over the capital inputs m that are most technologically relevant for production in j and k.

In Proposition 1, I formalize this intuition by log-differentiating equation (8). First, the impact of a demand shock in industry k on firm sales in the same industry k is always positive. But the impact of a shock in k on sales in another industry j is ambiguous. Spillovers are more positive and stronger among industries that share more scalable (high ρ_m) and mutable (low θ_m) capital inputs.

Proposition 1 (Spillovers within the Firm) *The elasticity of expected firm sales in any industry j, $\mathbb{E}[X_{fj}]$, to a change in residual profitability in any industry k is given by:*

$$\psi_{fjk} \equiv \frac{\mathrm{d}\log \mathbb{E}[X_{fj}]}{\mathrm{d}\log \xi_{fk} B_k} = \sum_{m\in\mathcal{M}} (\rho_m - \theta_m)\,\lambda_{fjm}\,\mu_{fmk} + \mathbf{1}_{j=k} \sum_{m\in\mathcal{M}} \theta_m\,\lambda_{fjm}, \quad \forall j,k \in \mathcal{J}, \quad (11)$$

where μ_{fmk} are deployment shares given by equation 7 and λ_{fjm} denote utilization shares: the share of gross profits of industry j attributable to potential contributions by capital input m:

$$\lambda_{fjm} \equiv \frac{\mu_{fmj}\Delta_{fm}^{\rho_m} w^{1-\rho_m}}{\sum_{m'} \mu_{fm'j}\Delta_{fm'}^{\rho_{m'}} w^{1-\rho_{m'}}}.$$

Proposition 1 rationalizes the existence of both positive and negative cross-industry elasticities of sales to demand shocks in Section 1. The theory also nests two instances where the null hypothesis of production independence (zero spillovers) would hold. First, production is independent when capital inputs are industry-specific, i.e. each input m affects only a single industry j. Second, production can also be independent under the knife's edge case where scale and substitution effects perfectly offset: $\theta_m = \rho_m\ \forall m$. For example, if capital inputs are completely unscalable and also perfectly mutable ($\rho_m = 1 = \theta_m$), capital accumulation is isomorphic in expectation to exogenous firm-industry 'productivity draws'.[34]

The cross-industry (expected) sales elasticities in Proposition 1 can be decomposed into

of θ in the Fréchet functional form for expected value.

[34] Technically, when $\theta = 1$, deployment shares μ_{fmj} still respond to changes in market profitability across industries $\{\delta_{fmj}\}_{k\in\mathcal{J}}$. However, the value of capital *conditional* being deployed to a particular industry changes as well. Like in Eaton and Kortum (2002), two forces generate perfect offset such that the ex-ante expected value of capital deployed to any industry is invariant to changes in market conditions.

constituent extensive and intensive margin responses. Equation (9) in Lemma 2 provides an analytical expression for the extensive margin entry probability in a given industry. The impact of demand shocks on the probability of entry depends on the same scale and mutability properties of capital inputs discussed in Proposition 1. Lemma 3 in the Theory Appendix provides an exact decomposition and shows that the total elasticity of sales to demand shocks mostly occurs on the extensive margin for small firms and mostly on the intensive margin for larger firms (such as those in my regression sample).

3 Structural Estimation

While the reduced-form results in Section 1 focus on intensive margin log-linear responses, the theory in Section 2 predicts non-linear functional forms and variable cross-industry elasticities. This section bridges the divide. I use an exact connection between moments in the data and the theory to structurally estimate the model's parameters. In line with the reduced-form evidence, I estimate that knowledge inputs are scalable as well as mutable, whereas other capital inputs are much less scalable and mutable.

3.1 Overview and Assumptions

I conduct inference through a nested-fixed-point search across micro and macro parameters. Conditional on macro variables (residual demand levels, technology parameters), I use cross-industry responses to shocks within the firm to identify micro parameters (scalability and mutability). Conditional on micro parameters, I use observable 'macro' data from the BEA (industry gross output and industry-by-input expenditures) to identify macro variables by inverting a non-linear system of equilibrium conditions. This subsection provides assumptions that enable the interface between micro and macro.

Input Taxonomy

Estimation relies on data on industry-by-input expenditures from the BEA Industry accounts. I classify stage II production inputs as the set of agriculture, mining, construction, utilities, manufactures, wholesale, retail and transportation industries, and labor value added.[35] I assume that the production input composite l_{fj} in each industry is a homothetic constant-returns aggregate over these inputs.

[35] These correspond to NAICS sectors 1, 2, 3, 4, and labor value added.

I group remaining inputs that are less directly related to production into four categories of 'capital' inputs denoted by $m \in \mathcal{M}$. Consistent with the reduced-form evidence in Figure 2, I use the same three categories of knowledge-generating inputs that plausibly contribute towards the firm's intangible capital stock: (i) leasing of intangibles, (ii) headquarters services, and (iii) information and professional services. Finally, I create (iv) a residual capital input category containing all remaining inputs delineated in the input-output tables.[36]

I estimate a pair of scale and mutability parameters $(\rho^{KLG}, \theta^{KLG})$ common to the first three categories of knowledge inputs, and another set of scale and mutability parameters $(\rho^{RES}, \theta^{RES})$ for the residual capital input. I let $\Theta \equiv \{\rho^{KLG}, \theta^{KLG}, \rho^{RES}, \theta^{RES}\}$ denote the parameter set containing these production function elasticities.

Variation over time

To leverage time-variation within the firm for structural estimation, I interpret data from each period $t = \{1, 2, 3\}$ as the outcome of a separate static equilibrium featuring the two stages of capital accumulation and production. This is equivalent to assuming that accumulated capital depreciates completely at five-year frequencies and so all inputs of the firm are variable.[37] However, latent firm-industry profitability shifters, ξ_{fjt}, persist over time. These attributes stand in for fixed capital and explain correlation of outcomes within a firm over time even in the absence of other shocks.

I fix the overall mass of potential entrants at N. These firms naturally move in and out of industries over time due to stochasticity in the accumulation of capital. An inactive firm is any firm that, despite its chosen level of capital expenditures, has not had any arrivals of capital. In any period in which this happens, the firm will have zero sales, fall out of the observed sample, and thus 'exit'. Likewise, in any subsequent period in which the same firm accumulates capital, the firm reappears as an entrant.

[36] These correspond to NAICS sectors 52, 531, 532, 56, 6, 7, 8, 9 and physical capital investments. I collapse all these inputs to just one category because the reduced-form results do not uncover any bilateral expenditure-driven industry proximity other than knowledge inputs (the first three categories) that generates spillovers. This significantly reduces the number of technology parameters that I need to invert for and speeds up computation.

[37] The memoryless and additive separability assumptions in the model enable tractability even in a full-fledged dynamic setting. Estimation of depreciation parameters, however, would require longer periods of data than what I have.

Profitability Shifters and Demand Shocks

The firm's profitability shifters across industries, ξ_{fjt}, are unobservable to the econometrician. I assume that they are distributed according to a joint lognormal:

Assumption 3 (Profitability Shifters are distributed Lognormal) *Latent firm-industry profitability shifters are distributed joint lognormal according to:*

$$\xi_{fj,t=1} = \zeta_{fj}\ \zeta_f, \qquad \forall f, j$$
$$\log \zeta_{fj} \sim_{i.i.d.} \mathcal{N}(0, \gamma_0), \qquad \log \zeta_f \sim_{i.i.d.} \mathcal{N}(0, \gamma_1), \qquad \forall f, j.$$

The means of the lognormal distributions are isomorphic to proportional shifters in industry residual demand, B_j, so the normalization to 0 comes without loss of generality. The variance parameters $\boldsymbol{\gamma} \equiv \{\gamma_0, \gamma_1\}$ control firm-level comparative and absolute advantage respectively. First, the higher is γ_0, the more dispersed is profitability across industries within the firm, and the more persistent is the firm's extensive margin across periods (due to firms repeatedly accumulating capital in its most profitable industries). I estimate γ_0 by matching the aggregate share of industries of multi-industry firms that survive over 5-year intervals to that in the data, equal to 0.42. Second, the higher is γ_1, the larger are differences in absolute firm size. I estimate γ_1 by matching the aggregate share of sales by multi-industry firms in 1997 to that in the data, equal to 0.75.

In each year, a discrete and measure-zero set of firms $\mathcal{F}^{\mathcal{T}}$ receive export demand shocks $\{\Delta \log S_{fjt}\}_{j \in \mathcal{J}}$ as constructed in the data. I assume that these shocks affect firm profitability, $\{\xi_{fjt}\}_{j \in \mathcal{J}}$ as follows:

Assumption 4 (Impact of Demand Shocks) *Demand shocks as constructed in the data affect firm-industry latent profitability shifters according to*

$$\Delta \log \xi_{fj,t} = \nu\ \Delta \log S_{fjt}, \qquad \forall f \in \mathcal{F}^{\mathcal{T}},\ j \in \mathcal{J},\ t = \{2, 3\}.$$

The parameter ν is a 'first-stage' elasticity that captures the impact of an export demand shock $\Delta \log S_{fkt}$ at shifting a firm's profitability in that industry.[38] I estimate ν by relying on the following log-linear relationship between expenses on professional services (one of the four categories of capital inputs), M_{ft}^{PROF}, and demand shifters at the level of the

[38] This assumption can be explicitly micro-founded in a multi-destination exporting model in which firms draw different latent initial taste shifters across destinations. These initial shifters inform pre-existing patterns of exporting, and subsequent changes in foreign market size across destinations will differentially affect firms' market sizes.

firm:

$$\Delta \log M_{ft}^{PROF} = \rho^{KLG} \cdot \nu \sum_{k \in \mathcal{J}} \eta_{fk,t-1}^{PROF} \Delta \log S_{fkt},$$

where the theoretical expenditure shares $\eta_{fk,t-1}^{PROF}$ are approximated using aggregate I/O expenditure shares of an industry on professional services, $\beta_{k,PROF}$:

$$\eta_{fk,t-1}^{PROF} \equiv \frac{\sum_{m \in \mathcal{M}} \mu_{fmk,t-1} \Delta_{fm,t-1}^{\rho_m}}{\sum_{m \in \mathcal{M}} \Delta_{fm,t-1}^{\rho_m}} \approx \frac{\beta_{k,PROF} X_{fk,t-1}}{\sum_k \beta_{k,PROF} X_{fk,t-1}}.$$

Column (1) of Table 3 estimates the combined elasticity at $\rho^{KLG}\nu = 0.65$. This 'offline' relationship yields a value of ν conditional on any estimate of ρ^{KLG}, and helps reduce dimensionality of the non-linear search below.

3.2 Identification of Macro Variables

Table 4 provides an overview of the macro-level variables that can be identified conditional on micro parameters Θ, $\boldsymbol{\gamma}$, and ν. First, I set the mass of firms at $N = 318000$, the total number of unique firms ever to appear in the 1997 census of manufacturing (including administrative and inactive records). Second, I read off ς_j from the share of gross output expensed on production inputs in the BEA industry tables. Third, I estimate the average arrival rate of capital, Z_t, by matching the model's share of multi-industry firms to that in the data in each year (0.2).[39]

Finally, I use BEA data on output X_{jt} and input expenditures M_{jmt} by industry to identify residual profitability indices B_j and technology matrix $\boldsymbol{\alpha} \equiv \{\alpha_{mj}\}_{m,j}$. I use data from 1997 (the latest year with detailed BEA input-output and capital use data) to identify $\boldsymbol{\alpha}$ and assume that these technology coefficients are time-invariant over the period of my sample. Although capital input expenditures in the model are at the firm-level, BEA data has these expenditures aggregated by industry. I assume that, for multi-industry firms, accounting over these expenditures is proportionalized across industries with respect to deployment shares μ_{fmj}.[40] I tally up the model's predicted industry sales and input

[39] This parameter mediates the importance of the extensive margin number of arrivals relative to the intensive margin quality per arrival. While it has no aggregate implications (aggregation includes both intensive and extensive margins), it does affect estimation because computation of the conditional moment condition matches firms in the model to that in the data based on their extensive margin outcomes.

[40] An alternative assumption is that the BEA allocates all of a firm's capital input expenditures to the firm's first industry of deployment. This yields the same aggregate prediction as equation (12).

Table 4: Overview of Model Primitives and Source of Identification

	Variable and Description	Source of Identification
Micro Parameters		
Θ	Scale and rivalry elasticities of capital inputs	Within-firm spillovers, ψ_{fjk}
γ_0	Within-firm heterogeneity in ζ_{fj}	Share of industries that continue (0.42)
γ_1	Across-firm heterogeneity in ζ_f	Share of sales by multi-industry firms (0.75)
ν	Responsiveness of ξ_{fjt} to export demand shocks	Assumption 4, Table 3, $\rho^{INT}\nu = 0.65$
Macro Variables		
N	Mass of latent firms	All active and inactive firms (318,000)
ς_j	Production input expenditures as a share of sales	Corresponding share in I/O Table
Z_t	Efficiency of innovation on the extensive margin	Share of multi-industry firms (0.2)
B_{jt}	Equilibrium industry-wide residual profit shifter	Gross Output X_{jt}
α_{mj}	Effectiveness of capital input m for industry j	I/O Table Expenditures M_{mj}

spending to yield the following system of $|J|\times 3+|J|\times|M|$ equations in as many variables:

$$\begin{aligned} N\int\sum_{m\in\mathcal{M}}\delta_{fmjt}^{\theta_m}\Delta_{fmt}^{\rho_m-\theta_m}\,\mathrm{d}G(\xi) &= (1-\varsigma_j)X_{jt}, \qquad \forall j\in\mathcal{J},\;\; t=\{1,2,3\}, \\ \frac{\rho_m-1}{\rho_m}N\int\delta_{fmjt}^{\theta_m}\Delta_{fmt}^{\rho_m-\theta_m}\,\mathrm{d}G(\xi) &= M_{mjt}, \qquad \forall j\in\mathcal{J},\;\; t=1,\;\; m\in\mathcal{M}, \end{aligned} \tag{12}$$

where $G(\xi;\boldsymbol{\gamma})$ is the joint-lognormal distribution of ξ given by Assumption 3.[41] I normalize the single factor price w_t to equal one in each year after deflating outcome variables by wage inflation. I exploit a recursive computational algorithm that speeds up inversion of this large set of macro variables and provide more details in Appendix C.1.

Notably, inference of macro variables is agnostic to general equilibrium details of the model (such as trade or input-output linkages) as long as they affect all firms equally. For example, if there is no selection into exporting, sourcing, or integration strategies, B_{jt} encapsulates the net effect of all export market access, import market competition, as well as prices of intermediate inputs. Put differently, while these forces affect the interpretation of what goes inside residual demand B_{jt}, they don't affect the inversion of B_{jt} outlined in equation (12). Data on sales X_{jt} is a sufficient statistic. Estimation is also invariant to values of returns to scale in the index of physical inputs γ_j and demand elasticities σ_j. Instead, the relevant parameter $\varsigma_j = \gamma_j\frac{\sigma_j-1}{\sigma_j}$ combines both and can be read off of the input-output tables.

[41] With an abuse of notation, a variable subscripted with f indicates that it is dependent on the vector of profitability shifters ξ_f.

3.3 Identification of Scale and Mutability Parameters

I define firm-industry level structural residuals, ϵ_{fjt}, as the linear deviation of observed sales in the data from expected sales in the model:

$$\epsilon_{fjt} \equiv X_{fjt} - \mathbb{E}[X_{fjt} \mid \xi_{ft}, B_t, \boldsymbol{\alpha}, Z; \Theta].$$

Variation in ϵ_{fjt} comes from stochasticity in the accumulation of capital φ_{fj} (driven by the Poisson arrival process and Fréchet-distributed match-specific shocks). The structural residuals are mean-independent of demand shocks $\Delta \log S_{fs}$ for any year s and industry presence in any prior year $\chi_{f,t-1}$, since the conditioning variables contain all the relevant information for the firm's expenditure and production decisions in any given year t. Formally, applying the law of iterated expectations (on ξ_{ft}) yields

$$\mathbf{E}[\epsilon_{fjt} \mid \Delta \log S_{fs}, \chi_{f,t-1}] = 0, \qquad \forall f, j, t, s.$$

I use the vector of demand shocks, $\Delta \log S_{fs}$, as instruments to identify Θ. However, a key challenge is that firm-specific profitability shifters ξ_{ft} are unobservable to the econometrician but plausibly correlated with next-period demand shocks. This correlation occurs mechanically in my empirical setting, where, by construction, demand shocks are zero whenever a firm is not active in an industry.

My identifying assumption is that demand shocks $\Delta \log S_{fs}$ are uncorrelated with unobserved profitability shifters, $\xi_{f,s-1}$ and sales $X_{f,s-1}$ *conditional* on pre-period industry presence, $\chi_{f,s-1}$. This assumption is a significant weakening over a simple (unconditional) exogeneity assumption, because it allows shocks to be non-parametrically correlated with past industry presence. In other words, identification only requires that shocks are as good as randomly assigned among firms with identical extensive margins.

Assumption 5 (Conditional Independence) *Export demand shifters are randomly assigned to firms conditional on pre-existing industry presence:*

$$\Delta \log S_{fjt} \perp \{\xi_{fk,t-1}, X_{fk,t-1}\}_{k \in J} \mid \{\chi_{fk,t-1}\}_{k \in J} \qquad \forall j \in \mathcal{J}, t = 2, 3.$$

I conduct inference using a moment estimator on a simulated sample of firms, $s \in S$. A computational challenge is that the profitability shifters ξ_f of firms in the data sample are unobserved. If demand shocks were purely exogenous, an average over the simulated firms drawn from the (prior) distribution of ξ_f given by Assumption 3 would suffice. But because demand shocks are only assumed to be exogenous conditional on a firm's pre-period extensive margin, I need to integrate over the (posterior) distribution of ξ_f *conditional* on the firm's extensive margin. I exploit the closed-form expressions for firm entry into an industry (equation 9) to compute analytical expressions for these posterior weights, denoted ω_{fs}.

Proposition 2 (Inference of Scalability and Mutability) *Let $\Delta \widetilde{\log S}_{fkt}$ denote demeaned demand shocks by industry-year. Define the following analytical sample moment conditions for a pair of industries j, k and year $t = \{2, 3\}$:*

$$m_{jkt} \equiv \frac{1}{n_{jk,t-1}} \sum_{f \in n_{jk,t-1}} \left((X_{fjt} - X_{fj,t-1}) - \sum_{s \in S} \omega_{sf,t-1}\, \mu_{fjk}(\xi_s, \Delta \log S_{ft}) \right) \Delta \widetilde{\log S}_{fkt},$$

where $n_{jk,t-1}$ is the set of firms f in the data with positive sales in j and k in year $t-1$, $s \in S$ denotes a sample of simulated firms with profitability shifters ξ_s drawn from distribution $G(\xi_s)$ according to Assumption 3, ω_{fs} refers to the probability that a simulated firm (s) (with fundamentals ξ_s) matches that of a firm (f) in the data (industry presence $\chi_{f,t-1}$) relative to other $s' \in S$:

$$\omega_{fs,t-1} \equiv \frac{\prod_j Pr(\chi_{j,t-1} = \chi_{fj,t-1} | \xi_s)}{\sum_{s'} \prod_j Pr(\chi_{j,t-1} = \chi_{fj,t-1} | \xi_{s'})},$$

and μ_{fjk} is model-implied expected sales growth given by

$$\mu_{fjk} = \mathbb{E}[X_{fjt} | \xi'_s] - \mathbb{E}[X_{fj,t-1} | \xi_s, \chi_{fj,t-1} = 1],$$

where next-period latent profitability ξ'_s evolves conditional on ξ_s and empirical demand shocks $\Delta \log S_{ft}$ according to Assumption 4.

At true parameter values Θ, γ, as the data and simulation samples get large, $N, S \to \infty$, the sample moment $m_{jkt} = 0$ for any j, k and $t \in \{2, 3\}$.

Proposition 2 delivers $J \times J$ moment conditions (for each year $t = 2, 3$) that can be used to estimate Θ. In practice, there are very few non-zero observations for each pair of industries j and k. I thus collapse the moments in each year to the following four categories based on ex-ante observable bilateral characteristics.

(i) Main diagonals of the matrix, $j = k$, for industries j where expenditures on knowledge inputs as a share of gross output is higher than the mean. This helps identify the scalability of knowledge inputs ρ^{KLG}.

(ii) Main diagonals of the matrix, $j = k$, for industries j where expenditures on knowledge inputs as a share of gross output is lower than the mean. This helps identify the scalability of residual capital inputs ρ^{RES}.

(iii) Off-diagonals of the matrix, $j \neq k$, for industry pairs jk whose proximity in the use of knowledge inputs is higher than the mean over all industry pairs. This helps identify the mutability of knowledge inputs θ^{KLG}.

(iv) Off-diagonals of the matrix, $j \neq k$, for industry pairs jk whose proximity in the use of knowledge inputs is lower than the mean over all industry pairs. This helps identify the mutability of residual capital inputs θ^{RES}.

In total, I estimate six non-linear micro parameters $(\rho^{KLG}, \theta^{KLG}, \rho^{RES}, \theta^{RES}, \gamma_0, \gamma_1)$ using ten moments: four dynamic moments each year (for $t = 2, 3$) corresponding to the four groupings of cells in the $J \times J$ matrix to estimate Θ, and two remaining cross-sectional moments (from $t = 1$) used to estimate the variances in the lognormal distribution: γ_0, γ_1. I use the identity weighting matrix to weigh moments.[42] Estimation proceeds as follows:

1. Simulate a fixed set of 2000 firms, with baseline draws of ζ_{sj}, ζ_s from standard normal distributions. I use stratified sampling to over-weigh firms with higher ζ_s.

2. Guess a starting $\hat{\Theta}$, $\hat{\gamma}_0$, $\hat{\gamma}_1$, then repeat Steps 3-5 until convergence criterion is met.

3. Compute ξ_{sj} given $\hat{\gamma}_0$, $\hat{\gamma}_1$ from Assumption 3 and ζ_{sj}, ζ_s.

4. Given ξ_{sj} and $\hat{\Theta}$, use macro data to invert for α_{mj}, B_{jt}, Z via equation (12).

5. Compute the sample moment conditions in Proposition 2, stack the moments as described above, and use a bounded Nelder-Mead simplex search algorithm to adjust the guess of $\hat{\Theta}$, $\hat{\gamma}_0$, $\hat{\gamma}_1$ given the change in the objective value.

Table 5 presents estimates of $\Theta, \boldsymbol{\gamma}$. Consistent with the reduced-form spillovers in Section 1, I find that both sets of knowledge and residual capital inputs are mutable (with low θ). However, knowledge inputs differ in that they are much more scalable within the firm. The differences are large enough to generate opposing forces for spillovers: industry pairs that share more knowledge inputs will have stronger economies of scope.

[42]Estimates do not change by much when using the optimal weighting matrix under 2-step GMM, which I compute when conducting the test of over-identifying restrictions.

Table 5: Estimated Scale, Mutability, and Firm Heterogeneity Parameters

Parameter	Description	Estimate	S.E.
ρ^{KLG}	Knowledge Input Scalability	12.64	(0.39)
θ^{KLG}	Knowledge Input (inverse) Mutability	3.61	(0.07)
ρ^{RES}	Residual Input Scalability	2.63	(0.05)
θ^{RES}	Residual Input (inverse) Mutability	4.06	(0.12)
γ_0	Degree of comparative advantage within the firm	0.85	(0.03)
γ_1	Variation in absolute advantage across firms	0.99	(0.05)
Test of Over-identifying Restrictions: $7.28 \sim \chi^2_4$			$p = 0.12$

Notes: This table reports estimates of micro parameters in the model. The six parameters are estimated on the sample of all multi-industry firms and the industries in which they are active, in years 2002-2007 and 1997-2002, using 10 moments. There are 13,000 (rounded) firm-year observations used in the sample. Standard errors of estimates are computed based on results from 21 bootstrap samples, where I re-draw over both the data and the simulated ξ samples.

3.4 Model Fit and External Validation

The estimated parameters fit the micro variation targeted in the data reasonably well. A test of over-identifying restrictions does not reject the null that the moments used in estimation are jointly valid. In the rest of this section, I provide external validation. Despite its limited number of (six) micro parameters, the model is capable of reproducing static moments of the data that are not targeted in estimation.

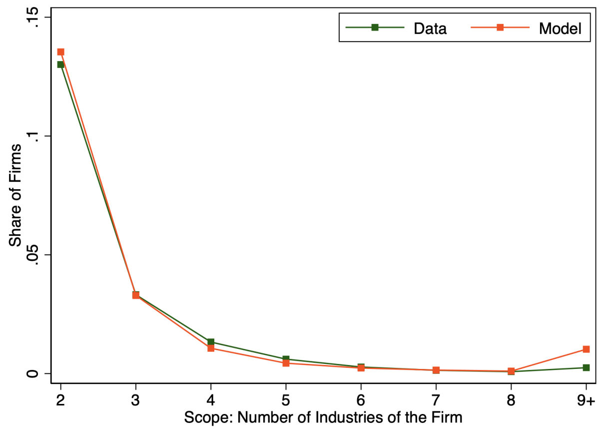
First, the model matches the distribution of the number of firms and their sales over firm scope. Figure 4 shows that the data and the model form a close match—firms with higher scope are increasingly scarce but also increasing large in overall sales. Both the data and the model attribute a significant size premium to the extreme right tail of the firm scope distribution (firms with 9 or more industries), though the model somewhat undershoots the data because it cannot account for the existence of true conglomerates and holding companies. Overall, the close fit between the model and data validate the model's parametric assumptions (Poisson and Fréchet).

Lastly, I test the model's predictions on *which* industries tend to be jointly produced against the data. I measure joint production in a pair of industries j, k as the share of industry j sales by firms that also sell in k:

$$JointProd_{jk} \equiv \frac{\sum_f X_{fj} \mathbf{1}_{X_{fk}>0}}{\sum_f X_{fj}},$$

Figure 4: External Validity: Distribution of Firms and Sales by Scope

(a) Share of firms by scope (b) Share of sales by scope



Notes: Panel (a) plots the share of total firms accounted for by firms across the scope distribution, comparing the data (in 1997) to the model. Panel (b) plots the share of total sales accounted for by firms across the scope distribution, comparing the data to the model. See Appendix Table 12 for the numbers behind these figures.

and industry joint-utilization of knowledge inputs as

$$Prox_{jk}^{KLG} \equiv \sum_{m \in \mathcal{M}^{KLG}} \beta_{mj} \frac{\beta_{mk}}{\sum_{k'} \beta_{mk'}}.$$

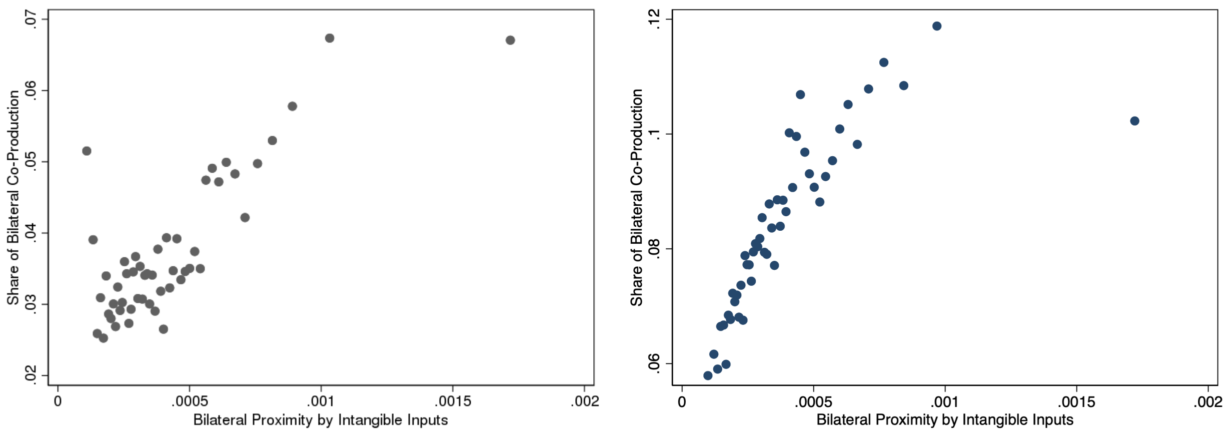
Empirically, joint production between a pair of industries is higher the more that the pair of industries share knowledge inputs. In panel (a) of Figure 5, I visualize this relationship in the data in terms of a binscatter for the full 42,240 pairs of industries $j \neq k$ in 1997. In panel (b), I find that the estimated model reproduces this strong bilateral positive association between joint production and the joint utilization of knowledge inputs. A firm in the model is more likely to jointly enter a pair of industries j, k the more that the industries share knowledge inputs. Intuitively, joint entry is more likely to occur because knowledge inputs are more scalable (resulting in higher investments by the firm) and mutable (resulting in a greater fraction of times that the firm chooses to accumulate capital in both industries).

4 The Macroeconomic Implications of Joint Production

Joint production within the firm leads to aggregate economies of scale and scope—a new dimension through which industries are linked in the macroeconomy. I characterize

Figure 5: External Validity: Joint production in the Data and the Model

(a) Joint production in the Data (b) Joint production in the Estimated Model



Notes: These panels display the relationship between bilateral industry joint production (share of industry j sales by firms with activities in k) and a measure of jk proximity in the utilization of knowledge inputs, in (a) the data, and (b) the model.

these effects using a matrix, $\mathbf{\Psi}$, of cross-industry elasticities of the producer price index to market size. I show how elements of this macro cross-industry matrix depend on the microeconomic properties of joint production—scalability and mutability of capital inputs. Using the estimated production parameters, I quantify the mechanism in general equilibrium and apply the model to revisit the implications of US trade policy.

4.1 Industry Linkages from Joint Production

I close the model in general equilibrium by limiting cross-industry effects to either internal properties of joint production or external input-output linkages. I shut down demand-side industry linkages by assuming that upper-tier consumer demand is Cobb-Douglas across industries. I provide results under two alternative assumptions on the production input composite l_{fj}: (i) that it is purely labor value-added, so there are no vertical production linkages, or (ii) that it is a Cobb-Douglas constant-returns aggregator over production inputs (which include the output of upstream manufacturing industries).

I model the US economy (denoted u) under trade with a set of foreign partners (denoted $d \in \mathcal{D}^F$) and assume that foreign variables (industry expenditures and price indices) are exogenous. Domestic price indices, however, are endogenous and feed back to firm production decisions through the usual input price and output competition (residual profitability) mechanisms. I abstract from wage effects by assuming that there is a large enough non-manufacturing sector that is traded at some exogenous world price. Rather than causing domestic wages to adjust, trade policies and foreign shocks affect US total manufacturing output. I abstract from the formation of new fixed capital (i.e. new draws of $\{\xi_{fj}\}_f$ through firm creation) and focus on interactions among existing (including inactive) firms. All firm profits are spent on the non-manufacturing sector to shut out manufacturing-demand-driven feedback effects.

Definition 1 in the Quantitative Appendix formalizes these assumptions in terms of general equilibrium conditions in the economy. I study the impact of exogenous demand shocks on the domestic (quality-adjusted) producer price index (PPI), $\mathcal{P}_j$:

$$\mathcal{P}_j^{1-\sigma_j} \equiv N \int \mathbb{E}\left[p_{fj}^{1-\sigma_j}\right] \mathrm{d}G(\xi).$$

I log-differentiate the system of market clearing equations in Definition 1 to derive the general equilibrium impact on domestic producer price indices, $\mathcal{P}$, from any exogenous unit-elasticity shifter of market size, S.

Proposition 3 (Aggregate Impact of Joint Production) *Under the open economy equilibrium characterized in Definition 1, a vector of exogenous shocks to market size faced by domestic firms,* $\mathrm{d}\log S$*, generate the following effects on the vector of domestic producer price indices* $\mathrm{d}\log\mathcal{P}$*:*

$$\mathrm{d}\log\mathcal{P} = diag\left(\frac{1}{\sigma-1}\right)\left(\mathbb{I} - \mathbf{\Omega}^S diag(\lambda_u^M) + \mathbf{\Psi}\left(\mathbb{I}-\mathbf{\Omega}^D\right)^{-1} \; diag(\lambda^{cpt})\right)^{-1} \times \\ \mathbf{\Psi}\left(\mathbb{I}-\mathbf{\Omega}^D\right)^{-1} \mathrm{d}\log S, \quad (13)$$

where $\mathbf{\Psi}$ is a $J \times J$ is an inverse matrix of cross-industry supply elasticities $\mathbf{\Upsilon}$ for a fictitious 'average' firm:

$$[\mathbf{\Psi}]_{jk} \equiv \sigma_j(1-\varsigma_j)[\mathbf{\Upsilon}^{-1}]_{jk} - \mathbf{1}_{j=k},$$
$$[\mathbf{\Upsilon}]_{jk} \equiv \sum_m (\rho_m - \theta_m)\,\bar{\lambda}_{jm}\,\bar{\mu}_{jmk} + \mathbf{1}_{j=k}\sum_m \theta_m\,\bar{\lambda}_{jm}, \qquad \forall j,k \in \mathcal{J},$$

where $\bar{\lambda}$ and $\bar{\mu}$ are weighted averages over firm-level utilization and deployment shares:

(i) Deployment shares $\bar{\mu}_{jmk}$ indicate the economy-wide propensity for capital of type m to be deployed in industry k (relative to other industries k') among firms that produce in j:

$$\bar{\mu}_{jmk} \equiv \int \frac{\mathbb{E}[X_{fj}]\lambda_{fjm}}{\int \mathbb{E}[X_{fj}]\lambda_{fjm}\,\mathrm{d}G(\xi)}\mu_{fmk}\,\mathrm{d}G(\xi),$$

(ii) Utilization shares $\bar{\lambda}_{jm}$ indicate the aggregate contribution to industry j of capital generated by input m (relative to other capital inputs m'):

$$\bar{\lambda}_{jm} \equiv \int \frac{\mathbb{E}[X_{fj}]}{\int \mathbb{E}[X_{fj}]\, \mathrm{d}G(\xi)} \lambda_{fjm}\, \mathrm{d}G(\xi),$$

$\mathbb{I}$ is the identity matrix, and $\mathbf{\Omega}^S$, $\mathbf{\Omega}^D$ are matrices containing external input-to-output coefficients that depend on the share of industry k gross output on production inputs from industry j, γ_{kj}:

$$[\mathbf{\Omega}^S]_{jk} \equiv \beta_{jk} \frac{\sigma_j}{\sigma_k - 1}, \qquad [\mathbf{\Omega}^D]_{jk} \equiv \lambda^X_{uj}(1 - \lambda^X_{j,F}) \frac{\beta_{kj} X_k}{\sum_{k' \in \mathcal{J}} \beta_{k'j} X_{k'}},$$

$\lambda^X_{j,F}$ is the share of final use among all consumption in industry j, λ^X_{dj} is the share of the home country's sales going to d, λ^M_{dj} is the share of country d's consumption originating from the home country, and λ^{cpt}_j measures the potential for home-country firms to gain market share from foreign competitors in each market:

$$\lambda^{cpt}_j \equiv \sum_{d \in \{u, \mathcal{D}^F\}} \lambda^X_{dj}(1 - \lambda^M_{dj}).$$

Equation (13) highlights three forces behind the equilibrium impact of demand shocks on industry PPI. First, input-output linkages transmit changes in the output of any one industry into changes in demand for another (through $\mathbf{\Omega}^D$), and transmit changes in the price index of any one industry into cost shocks in another (through $\mathbf{\Omega}^S$). Second, the open-economy market-stealing term (λ^{cpt}_j) measures whether changes in producer prices allow domestic firms to win market share against foreign producers. Third, joint production within the firm delivers a matrix of aggregate cross-price elasticities of supply ($\mathbf{\Psi}$), channeling profit opportunities in one industry into capital accumulation in another.

While the first two forces are familiar, this third channel—joint production—is new. The joint production matrix $\mathbf{\Psi}$ encodes the role of input scalability and mutability behind the supply-side response of prices to output. This mechanism exists irrespective of general equilibrium factor price and input price changes (such as those from Heckscher-Ohlin models or input-output linkages).

For intuition on the role of the joint production matrix, $\mathbf{\Psi}$, consider a simple economy in autarky ($\lambda^{cpt}_j = 0$) without production input-output linkages ($\mathbf{\Omega}^P = \mathbf{\Omega}^R = 0$). With the first two channels shut down, the impact of demand shocks (equation 13) reduces to

$$\mathrm{diag}(\sigma - 1)\, \mathrm{d} \log \mathcal{P} = \mathbf{\Psi}\, \mathrm{d} \log S,$$

where the the off-diagonal elements of $\mathbf{\Psi}$ are zero if and only if $\sum_m (\rho_m - \theta_m)\bar{\lambda}_{jm}\bar{\mu}_{jmk} = 0$ for all pairs of industries j, k. The same two conditions that cause production to be independent across industries within the firm (discussed in Section 2.5) also shut down aggregate spillovers. This occurs if either (i) each industry relies on a different set of capital inputs (so that $\bar{\lambda}_{jm}\bar{\mu}_{jmk} = 0$), or (ii) scalability and mutability forces lie on a knife's edge (so $\rho_m = \theta_m \ \forall m$) that causes perfect offset.

The knife's-edge case of independent production across industries nests the predictions of most macro models. When $\mathbf{\Psi}$ is active only along the main diagonal, the elasticity of the industry price index responds only to own-industry demand shocks, so that in each industry j:

$$\mathrm{d}\log \mathcal{P}_j = \frac{1}{\sigma_j - 1}\left(\frac{\sigma_j(1-\varsigma_j)}{\sum_m \bar{\lambda}_{jm}\rho_m} - 1\right)\mathrm{d}\log S_j.$$

This elasticity, bounded between $(-\frac{1}{\sigma_j-1}, 1)$, reflects the strength of industry-level economies of scale. It is mediated by the degree of returns to scale in capital inputs, ρ_m, and production inputs, γ_j. Together, they generate industry supply curves that are either downward sloping, flat, or upward sloping, nesting predictions in recent models with variable own-industry returns to scale (Kucheryavyy et al., 2019; Bartelme et al., 2019).

Two limit cases are worth pointing out. As $\rho_m \to \infty$ for all m, the industry-level scale elasticity ($1/(\sigma_j - 1)$) is equivalent to that in a multi-industry Krugman (1980) model with industry-specific free entry. The strength of internal returns to scale in production approaches $1 + \frac{1}{\sigma_j-1}$ and is equivalent to the gains from variety $\frac{1}{\sigma_j-1}$ achieved by outside entrants in a constant-internal-returns Krugman model.[43]

Conversely, as ρ_m approaches the other limit of 1, capital inputs have no impact on production. Firms respond to market size shifters only by adjusting their use of production inputs, l_{fj}. The production input returns to scale γ_j thus shapes the response of the price index to market size shifters. If there are constant returns to scale behind production, $\gamma_j = 1$ and the entire joint production matrix shuts down, so that $\mathbf{\Psi} = 0$, indicating an absence of both economies of scope and scale. Intuitively, aggregate industry supply curves are flat so that demand shifters do not move prices.

4.2 Quantifying the Impact of Joint Production

I calibrate remaining parameters of the model to the US economy in 2017 and quantify aggregate returns to scale and scope from joint production. I model the US economy

[43] Assumption 2 limits the elasticity of output to accumulated capital at $\frac{\sigma_j-1}{\sigma_j} - \gamma_j$ so that the internal returns to scale cannot exceed the curvature of demand.

under trade with two foreign partners: China (singled out for the purposes of tariff counterfactuals), and a rest-of-the-world composite.

In this model of monopolistic competition, the CES demand elasticity (σ_j) affects the response of industry price indices to demand shocks. I calibrate $\sigma_j = 5$ for all industries j in the following quantitative exercises and explore sensitivity of results to alternative calibrations of σ_j in the Appendix.[44] I calibrate production input expenditure shares, β_{jk}, to match those observed in the I/O table under the assumption that the production input composite is a Cobb-Douglas bundle over inputs. Quantitative Appendix C.6 provides more details on the calibration of other parameters and model objects (e.g. exogenous foreign price indices) using aggregated industry-level production and trade data for the US in 2017.

I use the analytical propagation matrix in equation (13) to decompose the impact of a proportional increase in foreign market size (across all industries) on the US manufacturing PPI.[45] This is computed as a weighted average of industry-level producer price responses, with weights equal to current output shares across industries, λ_j^X:

$$\mathrm{d}\log PPI = \sum_{j \in \mathcal{J}} \lambda_j^X \, \mathrm{d}\log \mathcal{P}_j.$$

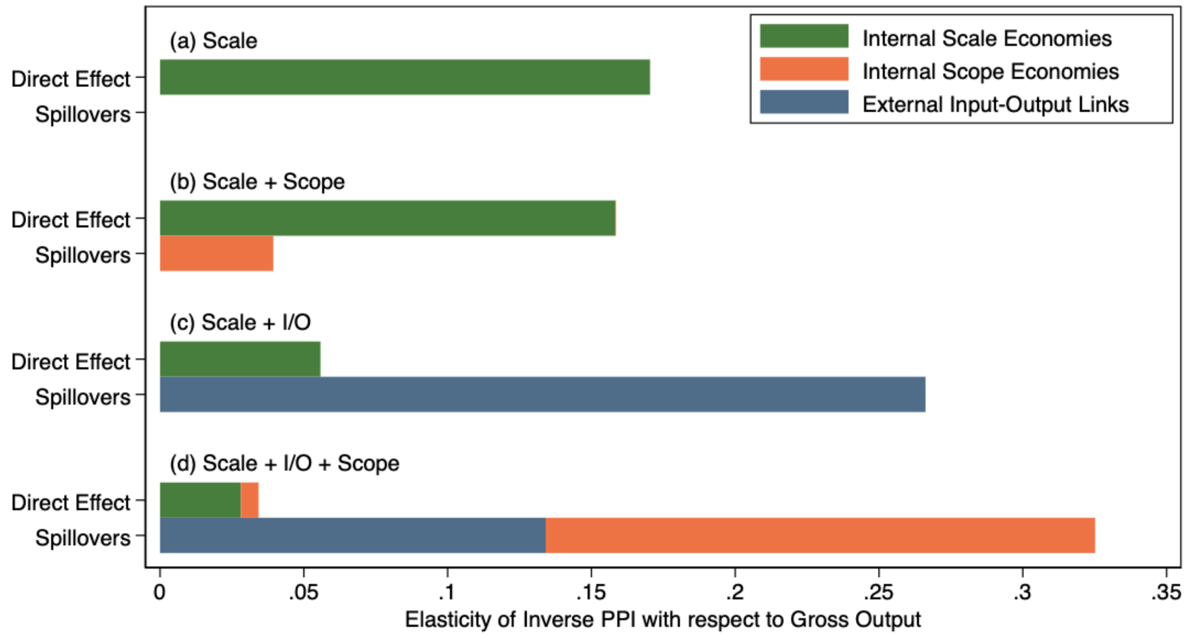
The net impact of foreign demand shocks on the manufacturing PPI can be decomposed by splitting up the matrix in equation (13) into (i) diagonal elements, representing the direct effect (of a shock in industry k on prices in the same industry k) versus (ii) off-diagonal elements, representing spillovers (a shock in k on prices in j).

Figure 6 illustrates results from performing the decomposition above under four scenarios, labelled (a)-(d). To impose appropriate units on the exercise, in each scenario I scale the price index change by the total response in gross output and multiply by -1, so that the sum over direct effects and spillovers yields the total elasticity of the inverse PPI to gross output. A positive elasticity indicates the presence of aggregate returns to scale or scope—prices decline with market size. The accompanying numbers (along with a companion decomposition for the response of gross output to foreign shocks) are given

[44] I don't estimate σ_j in this paper, and nor does identification of production-side elasticities in Section 3 depend on σ_j. Given Assumptions 1 and 2, estimation of input scalability and mutability depends on $\varsigma_j = \gamma_j(\sigma_j - 1)/\sigma_j$, which is a function of both the demand elasticity σ_j and returns to scale in production inputs γ_j. BEA data allow identification of ς_j but not σ_j separately from γ_j. Calibrating σ_j to a common value across industries is a reasonable benchmark for my counterfactuals because it ensures that the asymmetric industry results are not driven by differences in demand substitutability.

[45] The term $\mathrm{d}\log S$ refers to a unit-elasticity demand shifter for US producers. To scale a foreign shock $\mathrm{d}\log \bar{Y}_{dj}$ into a unit-elasticity impact I use $\mathrm{d}\log S = \lambda_{dj}^X \, \mathrm{d}\log \bar{Y}_{dj}$ as outlined in the proof the Proposition 3.

Figure 6: Elasticity of the Inverse PPI with respect to Gross Output



Notes: This figure decomposes the total elasticity of the US manufacturing PPI with respect to gross manufacturing output (induced by a proportional increase in foreign demand) into (i) a direct, own-industry effect (green bar), (ii) spillovers that accrue due to scope economies (orange bar) and (iii) spillovers that accrue due to input-output linkages (blue bar). This decomposition is performed under four different scenarios, (a)-(d), corresponding to different underlying assumptions internal joint production and external input-output links in the economy. For more details on the four scenarios, see the main text. Appendix Table 13 presents separately the responses of productivity and output used to compute these elasticities.

in Appendix Table 13.

Scenarios (a) and (b) treat the production input composite as labor value-added, while (c) and (d) feature input-output linkages among manufacturing production inputs. Scenario (a) assumes that an analyst has data only on own-industry price-elasticities of supply, (e.g. through estimating returns to scale using data on single-industry firms). In this scenario, because the off-diagonal elements of $\boldsymbol{\Upsilon}$ are zero, the only impact of foreign shocks on the economy are direct, own-industry responses (driven by internal economies of scale), depicted by the green bar in the figure.

Scenario (b), the benchmark case, incorporates the effect of joint production (so that all elements of $\boldsymbol{\Upsilon}$, and thus its inverse $\boldsymbol{\Psi}$, are potentially non-zero). The orange bar depicts price index responses accruing due to internal cross-industry spillovers (economies of scope). On net, scope economies generate an elasticity of producer prices to output of -0.04 and account for 20% of the total elasticity of the producer price index to gross output.[46]

Scenarios (c) and (d) of Figure 6 mirror the exercise in (a) and (b) but under a calibration featuring production input-output linkages across industries. Scenario (c) depicts these price responses in the presence of only own-industry returns to scale and external I/O linkages. Input-output linkages induce large propagation effects due to the resulting circular structure of production, depicted by the relative size of the blue bar. The total response of gross output is much higher under such a world. Because increased scale induces lower prices in each industry, the total response of the price index is now much greater, with a larger proportion explained by spillovers relative to the direct effects.

Finally, in scenario (d), I combine the effects of joint production ($\mathbf{\Psi}$) with input-output linkages ($\mathbf{\Omega}$). I find that internal production spillovers within the firm are quantitatively important not only in the setting without I/O links (comparing scenarios (a) and (b)) but also in the setting with external linkages (comparing scenarios (c) and (d)). Allowing for internal production spillovers more than doubles the total elasticity of the price index, almost all of which accrues due to cross-industry spillovers.

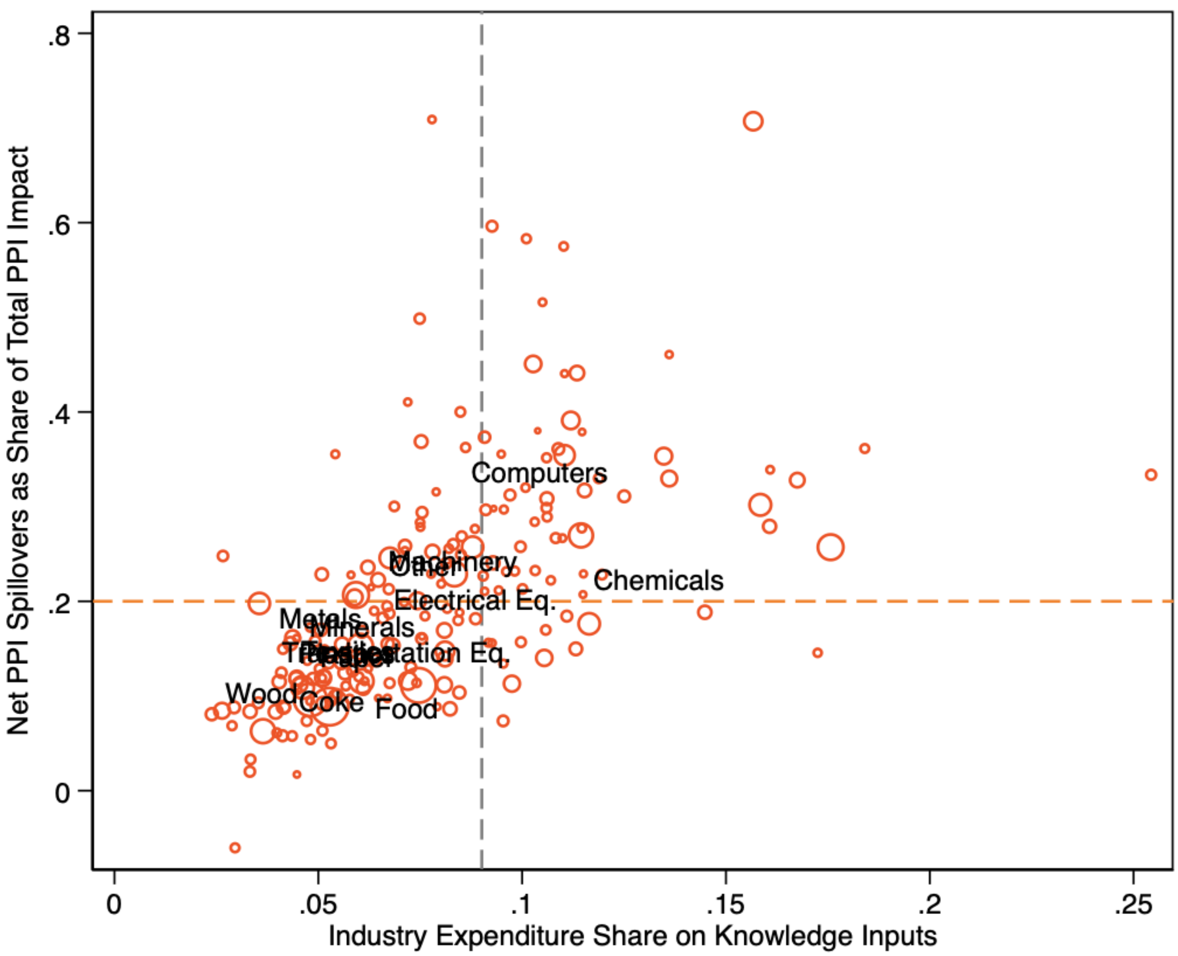
In addition to the large cross-industry responses on average, I find significant heterogeneity in the strength of production spillovers across industries. Knowledge-intensive industries are stronger transmitters as well as recipients. I decompose, for each industry, the impact of a foreign demand shock on aggregate PPI into (i) an own-industry price response and (ii) net price responses in other industries. Figure 7 plots the net cross-industry response as a share of the total PPI impact across the range of industries under scenario (b) (where economies of scale and scope are the only forces at play). This share ranges from 0.71 for totalizing fluid meter and counting device manufacturing to -0.06 for flavoring syrup.[47] Table 14 in the Quantitative Appendix presents a list of top and bottom industries.

Finally, I explore the sensitivity of results to alternative values of σ_j in Quantitative Appendix C.7.2. The key finding that spillovers due to joint production generates an equilibrium elasticity of PPI to gross output of -0.04 is insensitive across typical estimates of σ in the range of 3 to 10. I also allow σ_j to vary across industries by calibrating σ_j to match typical estimates of industry-level direct returns to scale or profit shares. Across both these calibrations, cross-price elasticities of supply due to joint production account for roughly 20 percent of the total price response.

[46]The direct effect (green bar) is slightly diluted owing to a higher response of gross output (the scaling on the denominator).

[47]Interestingly, flavoring syrup is the only industry in which negative spillovers (owing to residual capital inputs being less scalable and mutable) quantitatively overwhelm positive spillovers, causing a net negative price response in the event of a demand shock.

Figure 7: Knowledge-intensive Industries Drive Spillovers



Notes: This figure plots the net PPI spillover (as a share of the total PPI response) owing to an industry-level foreign demand shock, by industry expenditure share on knowledge inputs. The weighted-average spillover is given by the dashed line in orange and the weighted-average industry expenditure share is given by the dashed line in grey. Each circle represents a NAICS industry in proportion to its size. The computations are performed under scenario (b). Table 14 in the Quantitative Appendix presents a list of top and bottom industries. Overlaid in black text is the same statistic but aggregated to the level of broad 2-digit sectors. In order from the highest to lowest net spillovers, these sectors are: Computers, Machinery, Other, Chemicals, Electrical Equipment, Metals, Minerals, Textiles, Transportation Equipment, Plastic, Paper, Wood, Coke, and Food.

Table 6: Effects on the US Economy from US Tariffs on Chinese Imports

Model Import tariffs on Chinese imports	(1) Neoclassical CRS 20% Uniform	(2) Scale + Scope 20% Uniform	(3) Scale + Scope Alternative
Change (%)			
Consumer Price Index (CPI)	1.12	0.80	0.38
Producer Price Index (PPI)	0	-0.46	-0.67
Imports from China	-39.2	-40.5	-40.5
Imports from Rest of World	7.3	5.9	2.4
US Output	1.4	2.4	3.5
US Exports	0	2.1	2.8
Manufacturing Trade Deficit	-11.5	-19.3	-28.3
As Share of Manufacturing Output (%)			
New Manufacturing Profits	0	0.10	0.14
New Tariff Revenues	0.88	0.86	0.36

Notes: This table presents estimates of the impact of two different sets of tariffs on outcomes in the US economy, under two different model settings calibrated to match US national industry-level aggregates (industry output and trade patterns) in 2017. Column (1) presents results from a 20% increase in tariffs on Chinese imports across all industries, under a neoclassical model with constant returns to scale and perfect competition. Column (2) compares the impact of the same set of tariffs to my model with economies of scale and scope. Column (3) presents results from alternative tariffs identified by my model that minimize the CPI impact while achieving the same reduction in total imports from China. See Definition 1 for a characterization of the open economy equilibrium and Appendix Section C.8.3 for details on how I compute alternative tariffs.

4.3 Revisiting the Impact of Trade Protection

Given the findings of large and heterogeneous spillovers across industries, I apply the model to re-evaluate the impact of US tariffs on imports from China. In Table 6, I compute the impact on economy-wide aggregates (down the rows) of different tariff schedules and model environments (across the columns). I solve the system of equilibrium conditions in Definition 1 before and after the (large) counterfactual policy shock using 'exact hat algebra'. Quantitative Appendix C.8 provides details on this procedure.[48]

The first two columns of Table 6 display the aggregate impact of uniform tariffs on Chinese imports of 20% across all industries. In column (1), I calibrate a neoclassical trade model with constant returns to scale and perfect competition to the same US data in 2017, and compute the counterfactual price effects due to tariff policy. Under this benchmark, I

[48]To isolate the impact of my mechanism, I compute these results without taking into account production input-output linkages. The underlying economy corresponds to benchmark scenario (b) in Figure 13. See the Appendix Table 16 for results under input-output linkages and a discussion.

find that the CPI rises by 1.12% due to Chinese imports becoming more expensive.[49]

In comparison, column (2) applies my estimates of joint production and the full non-linear structure of the model to compute aggregate outcomes arising from the same tariff policy. Domestic US firms now benefit from reduced import competition, and prices of US goods fall not only in industries that are directly protected, but also in industries that are jointly produced. These endogenous responses of domestic prices (due to both scale and scope economies) substantially mitigate the price impact of import tariffs. The domestic PPI falls by 0.46%, lowering the overall rise in the CPI from 1.12% to 0.8%. Other margins of the US economy also improve due to increased competitiveness of US goods: exports go up, the deficit shrinks, and import substitution towards other countries is less pronounced.

Are there other tariff implementations that lower imports from China by the same amount (41%) at the cost of an even smaller rise in the CPI? While the optimal tariff structure is difficult to solve for in this non-linear environment, I use a similar local propagation matrix to that in equation (13) to compute the contribution of import tariffs, by industry, towards reducing Chinese imports (a 'benefit') and increasing the CPI (a 'cost'). I start with a baseline vector of tariff values that are biased towards industries with higher benefits per unit cost, and scale up the tariffs proportionately so that they generate the same counterfactual decline in imports from China.

The more biased is import protection towards knowledge-intensive industries, the smaller is the CPI-cost of reducing Chinese imports by 41%. When the degree of bias is such that tariffs on the most heavily affected industry reach 300%, the CPI rises by a mere 0.38%. Almost half of this reduction occurs as a result of a stronger decline in domestic producer prices, indicated by a greater reduction in the PPI of 0.67%. Column (3) of Table 6 reports the impact on US aggregate economic outcomes from this policy. Due to US producers lowering their prices (from increased capital accumulation), exports rise by 2.8%, the manufacturing trade deficit shrinks by 28.3%, and increased manufacturing net profits account for 0.14% of initial gross output.

Conclusion

This paper provides reduced-form evidence of joint production by assembling panel data on the industry-level sales of all US manufacturing firms and leveraging exogenous shocks

[49]The overall impact on welfare suggested by the results in column (1) is relatively low (especially after taking into account tariff revenue) and consistent with estimates in Amiti et al. (2019) and Fajgelbaum et al. (2019).

to their industry-specific market size. I find that a demand shock in one industry of a firm increases its sales in another the more that the pair of industries share knowledge inputs. I rationalize these results by developing a model of joint production under which inputs differ in their degree of scalability and mutability within the firm. I provide structural estimates of scale and mutability parameters and find that knowledge inputs generate economies of scope within firms: negative cross-industry price elasticities of supply.

Joint production generates a new and quantitatively important set of cross-industry linkages. The aggregate response of producer prices to market size would be roughly 20 percent lower if scope economies were not taken into account. Economies of scope are concentrated among industries that utilize more knowledge inputs, suggesting that these industries should be more closely scrutinized in quantitative studies and policy evaluation. In light of these findings, I re-examine the consequences of US tariffs on imports from China. I identify alternative tariff policies that more than halve the adverse CPI effects by biasing protection towards knowledge-intensive industries that feature stronger internal economies of scale and scope.

The results in this paper open up both micro and macro avenues for further research. At a micro level, the stochastic joint production framework can be applied to other imperfectly competitive settings featuring strategic interactions among firms. Richer firm-level data on research expenditures, patenting, levels of IT capital, and worker occupations could be used to understand how internal and external knowledge spillovers interact. The behavior of firms in the model also micro-found the endogenous accumulation of different types of capital across industries and countries. At a macro level, these results would improve our understanding of the origins, evolution, and correlation of comparative advantage across countries, as well as the welfare and productivity consequences of trade liberalization.

References

Acemoglu, Daron, Ufuk Akcigit, and William Kerr, "Networks and the Macroeconomy: An Empirical Exploration," *NBER Macroeconomics Annual*, jan 2016, *30* (1), 273–335.

_, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi, "The Network Origins of Aggregate Fluctuations," *Econometrica*, 2012, *80* (5), 1977–2016.

Aghion, Philippe, Antonin Bergeaud, Matthieu Lequien, and Marc J. Melitz, "The Heterogeneous Impact of Market Size on Innovation: Evidence from French Firm-Level Exports," *Mimeo*, 2019.

Almunia, Miguel, Pol Antràs, David Lopez-Rodriguez, and Eduardo Morales, "Venting Out: Exports during a Domestic Slump," *Mimeo*, 2018.

Amiti, Mary, Stephen Redding, and David Weinstein, "The Impact of the 2018 Trade War on U.S. Prices and Welfare," *Mimeo*, 2019.

Antràs, Pol, Teresa C. Fort, and Felix Tintelnot, "The Margins of Global Sourcing: Theory and Evidence from US Firms," *American Economic Review*, sep 2017, *107* (9), 2514–2564.

Arkolakis, Costas, "Market Penetration Costs and the New Consumers Margin in International Trade," *Journal of Political Economy*, dec 2010, *118* (6), 1151–1199.

_ and Fabian Eckert, "Combinatorial Discrete Choice," *Mimeo*, 2017.

_, Sharat Ganapati, and Marc-Andreas Muendler, "The Extensive Margin of Exporting Products: A Firm-level Analysis," *Mimeo*, 2019.

Armenter, Roc and Miklós Koren, "A Balls-and-Bins Model of Trade," *American Economic Review*, jul 2014, *104* (7), 2127–2151.

Atalay, Enghin, Ali Hortaçsu, and Chad Syverson, "Vertical Integration and Input Flows," *American Economic Review*, apr 2014, *104* (4), 1120–1148.

Aw, Bee Yan, Mark J Roberts, and Daniel Yi Xu, "R&D Investment, Exporting, and Productivity Dynamics," *American Economic Review*, jun 2011, *101* (4), 1312–1344.

Baqaee, David Rezza and Emmanuel Farhi, "The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem," *Econometrica*, 2019, *87* (4), 1155–1203.

Bartelme, Dominick, Arnaud Costinot, Dave Donaldson, and Andrés Rodríguez-Clare, "The Textbook Case for Industrial Policy: Theory Meets Data," *Mimeo*, 2019.

Baumol, William J, John C. Panzar, and Robert D. Willig, *Contestable markets and the theory of industry structure*, New York: Harcourt Brace Jovanovich, 1982.

Bernard, Andrew B, Emily J Blanchard, Ilke Van Beveren, and Hylke Vandenbussche, "Carry-Along Trade," *The Review of Economic Studies*, feb 2018, *86* (2), 526–563.

_, Stephen J Redding, and Peter K Schott, "Multiple-Product Firms and Product Switching," *American Economic Review*, mar 2010, *100* (1), 70–97.

Bilir, L. Kamran and Eduardo Morales, "Innovation in the Global Firm," *Journal of Political Economy*, jul 2019.

Bloom, Nicholas, Erik Brynjolfsson, Lucia Foster, Ron Jarmin, Megha Patnaik, Itay Saporta-Eksten, and John Van Reenen, "What Drives Differences in Management Practices?," *American Economic Review*, may 2019, *109* (5), 1648–1683.

—, Mark Schankerman, and John van Reenen, "Identifying Technology Spillovers and Product Market Rivalry," *Econometrica*, 2013, *81* (4), 1347–1393.

Boehm, Christoph E., Aaron Flaaen, and Nitya Pandalai-Nayar, "Input Linkages and the Transmission of Shocks: Firm-Level Evidence from the 2011 Tōhoku Earthquake," *The Review of Economics and Statistics*, mar 2019, *101* (1), 60–75.

Boehm, Johannes, Swati Dhingra, and John Morrow, "The Comparative Advantage of Firms," *Mimeo*, 2019.

Borusyak, Kirill, Peter Hull, and Xavier Jaravel, "Quasi-Experimental Shift-Share Research Designs," *Mimeo*, 2018.

Caliendo, L. and F. Parro, "Estimates of the Trade and Welfare Effects of NAFTA," *The Review of Economic Studies*, nov 2014, *82* (1), 1–44.

Costinot, Arnaud and Andrés Rodríguez-Clare, "Trade Theory with Numbers: Quantifying the Consequences of Globalization," in "Handbook of International Economics," Elsevier, 2014, pp. 197–261.

De Loecker, Jan, Pinelopi K. Goldberg, Amit K. Khandelwal, and Nina Pavcnik, "Prices, Markups, and Trade Reform," *Econometrica*, 2016, *84* (2), 445–510.

Desai, Mihir A, C. Fritz Foley, and James R Hines, "Domestic Effects of the Foreign Activities of US Multinationals," *American Economic Journal: Economic Policy*, jan 2009, *1* (1), 181–203.

Dhingra, Swati, "Trading Away Wide Brands for Cheap Brands," *American Economic Review*, oct 2013, *103* (6), 2554–2584.

Dhyne, Emmanuel, Amil Petrin, Valerie Smeets, and Frederic Warzynski, "Multi Product Firms, Import Competition, and the Evolution of Firm-product Technical Efficiencies," *Mimeo*, 2017.

di Giovanni, Julian, Andrei A. Levchenko, and Isabelle Mejean, "The Micro Origins of International Business-Cycle Comovement," *American Economic Review*, jan 2018, *108* (1), 82–108.

Eaton, Jonathan and Samuel Kortum, "Technology, Geography, and Trade," *Econometrica*, sep 2002, *70* (5), 1741–1779.

—, —, and Sebastian Sotelo, "International Trade: Linking Micro and Macro," in Daron Acemoglu, Manuel Arellano, and Eddie Dekel, eds., *Advances in Economics and Econometrics*, Cambridge University Press, 2013, pp. 329–370.

Eckel, Carsten and J. Peter Neary, "Multi-Product Firms and Flexible Manufacturing in the Global Economy," *Review of Economic Studies*, jun 2009, *77* (1), 188–217.

Ellison, Glenn, Edward L Glaeser, and William R Kerr, "What Causes Industry Agglomeration? Evidence from Coagglomeration Patterns," *American Economic Review*, jun 2010, *100* (3), 1195–1213.

Fajgelbaum, Pablo D., Pinelopi K. Goldberg, Patrick J. Kennedy, and Amit K. Khandelwal, "The Return to Protectionism," *Mimeo*, 2019.

Feenstra, Robert and Hong Ma, "Optimal Choice of Product Scope for Multiproduct Firms Under Monopolistic Competition," *Mimeo*, 2007.

Fort, Teresa C., "Technology and Production Fragmentation: Domestic versus Foreign Sourcing," *The Review of Economic Studies*, oct 2016, p. rdw057.

Färe, Rolf and Daniel Primont, *Multi-Output Production and Duality: Theory and Applications*, Springer Netherlands, 1995.

Gabaix, Xavier, "The Granular Origins of Aggregate Fluctuations," *Econometrica*, 2011, *79* (3), 733–772.

Garin, Andy and Felipe Silverio, "How Does Firm Performance Affect Wages? Evidence from Idiosyncratic Export Shocks," *Mimeo*, 2018.

Giroud, Xavier, "Proximity and Investment: Evidence from Plant-Level Data," *The Quarterly Journal of Economics*, mar 2013, *128* (2), 861–915.

_ **and Holger M. Mueller**, "Firms' Internal Networks and Local Economic Shocks," *American Economic Review*, oct 2019, *109* (9), 3617–3649.

Gort, Michael, *Diversification and Integration in American Industry*, Princeton University Press, 1962.

Grieco, Paul L. E. and Ryan C. McDevitt, "Productivity and Quality in Health Care: Evidence from the Dialysis Industry," *The Review of Economic Studies*, sep 2016, p. rdw042.

Griliches, Zvi, "Issues in Assessing the Contribution of Research and Development to Productivity Growth," *The Bell Journal of Economics*, 1979, *10* (1), 92.

Gumpert, Anna, Henrike Steimer, and Manfred Antoni, "Firm Organization with Multiple Establishments," *Mimeo*, 2019.

Haskel, Jonathan and Stian Westlake, *Capitalism without Capital*, Princeton University Press, nov 2017.

Hummels, David, Rasmus Jørgensen, Jakob Munch, and Chong Xiang, "The Wage Effects of Offshoring: Evidence from Danish Matched Worker-Firm Data," *American Economic Review*, jun 2014, *104* (6), 1597–1629.

Jia, Panle, "What Happens When Wal-Mart Comes to Town: An Empirical Analysis of the Discount Retailing Industry," *Econometrica*, 2008, *76* (6), 1263–1316.

Jovanovic, Boyan, "The Diversification of Production," *Brookings Papers on Economic Activity: Microeconomics 1993*, 1993.

Keller, Wolfgang and Stephen R Yeaple, "Multinational Enterprises, International Trade, and Productivity Growth: Firm-Level Evidence from the United States," *Review of Economics and Statistics*, nov 2009, *91* (4), 821–831.

Klette, Tor Jakob, "R&D, Scope Economies, and Plant Performance," *The RAND Journal of Economics*, 1996, *27* (3), 502.

— **and Samuel Kortum**, "Innovating Firms and Aggregate Innovation," *Journal of Political Economy*, oct 2004, *112* (5), 986–1018.

Krugman, Paul, "Scale Economies, Product Differentiation, and the Pattern of Trade," *American Economic Review*, 1980.

Kucheryavyy, Konstantin, Gary Lyn, and Andrés Rodríguez-Clare, "Grounded by Gravity: A Well-Behaved Trade Model with External Economies," *Mimeo*, 2019.

Lamont, Owen, "Cash Flow and Investment: Evidence from Internal Capital Markets," *The Journal of Finance*, mar 1997, *52* (1), 83–109.

Lashkari, Danial, Arthur Bauer, and Jocelyn Boussard, "Information Technology and Returns to Scale," *Mimro*, 2019.

Lim, Kevin, "Endogenous Production Networks and the Business Cycle," *Mimeo*, 2018.

Liu, Ernest, "Industrial Policies in Production Networks," *The Quarterly Journal of Economics*, aug 2019, *134* (4), 1883–1948.

Macedoni, Luca and Mingzhi Xu, "Flexibility and Productivity: Towards the Understanding of Firm Heterogeneity for Multi-product Exporters," *Mimeo*, 2019.

Mayer, Thierry, Marc J. Melitz, and Gianmarco I. P. Ottaviano, "Market Size, Competition, and the Product Mix of Exporters," *American Economic Review*, feb 2014, *104* (2), 495–536.

— **, Marc J Melitz, and Gianmarco I. P. Ottaviano**, "Product Mix and Firm Productivity Responses to Trade Competition," *Mimeo*, 2016.

McGrattan, Ellen R., "Intangible Capital and Measured Productivity," *Mimeo*, 2017.

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, nov 2003, *71* (6), 1695–1725.

Milgrom, Paul R. and John Roberts, "The Economics of Modern Manufacturing: Technology, Strategy, and Organization," *American Economic Review*, 1990.

Nocke, Volker and Stephen Yeaple, "Globalization and Multiproduct Firms," *International Economic Review*, oct 2014, *55* (4), 993–1018.

Orr, Scott, "Within-Firm Productivity Dispersion: Estimates and Implications," *Mimeo*, 2019.

Penrose, Edith Tilton, *The Theory of the Growth of the Firm*, Oxford: Blackwell, 1959.

Pierce, Justin R. and Peter K. Schott, "A concordance between ten-digit U.S. harmonized system codes and SIC/NAICS product classes and industries," *Journal of Economic and Social Measurement*, Oct 2012, *37* (1-2), 61–96.

Pokharel, Krishna P and Allen M Featherstone, "A Nonparametric Approach to Estimate Multiproduct and Product-specific Scale and Scope Economies for Agricultural Cooperatives," *Agricultural Economics*, 2019.

Rubin, Paul H., "The Expansion of Firms," *Journal of Political Economy*, jul 1973, *81* (4), 936–949.

Scharfstein, David S. and Jeremy C. Stein, "The Dark Side of Internal Capital Markets: Divisional Rent-Seeking and Inefficient Investment," *The Journal of Finance*, dec 2000, *55* (6), 2537–2564.

Schott, Peter K., "The relative sophistication of Chinese exports," *Economic Policy*, dec 2008, *23* (53), 5–49.

Shephard, Ronald William, *Theory of Cost and Production Functions*, Princeton University Press, 1953.

Stein, Jeremy C., "Internal Capital Markets and the Competition for Corporate Resources," *The Journal of Finance*, mar 1997, *52* (1), 111–133.

Tintelnot, Felix, "Global Production with Export Platforms," *The Quarterly Journal of Economics*, nov 2016, p. qjw037.

A Data Appendix

A.1 Data Construction and Details

Firms, Plants, and Products. I assemble data from the Economic Censuses (EC), the Longitudinal Business Database (LBD), and the Longitudinal Firm Trade Transactions Database (LFTTD) to construct a portrait of firm activity over the years 1997 to 2012. The Censuses are conducted quinquennially in years ending with '2' and '7'. Data on product shipments made by establishments come from the product trailer (PT) files which are attached to the Census of Manufactures (CMF). These trailer files contain responses of establishments that are sent a CMF 'Long Form'. The long form is sent to all establishments belonging to multi-establishments firms as well as a sample of single-establishment firms. The long form elicits shipments made by the establishment at a disaggregated level (varying from 6 to 10 digit NAICS).[50]

Using firm identifiers in the LBD, I match establishments to their parent firms and aggregate industry-level shipments from the level of the plant to the level of the firm. The firm identifier in the LBD comes from information the census collects on the span of control of firms in the Company Organization Survey and from tax identifier and plant identifier information in the Business Register. An establishment is a physical location where business activity occurs. The firm is defined (by the census) as the highest level entity that controls more than 50% of each of the establishments we assign to the firm. I drop plants that are administrative records (for which sales data are imputed).

External Sales. The CMF contains data on the shipments of a plant that go towards other plants within the same firm (i.e. inter-plant, intra-firm). However, this data is not broken down at the product-line level. For plants that produce in multiple industries, I apportion this inter-plant shipment data into industry-level intra-firm shipments using shares taken from the plant's total sales across industries. I then define the external sales of a firm in each industry as its total sales in that industry minus its intra-firm shipments. I drop external sales computed in this way in any industries of the firm that (i) account for less than 0.5% of firm-wide external shipments and (ii) are never the main produced industry of any plant the firm owns. This is conservative and allows product shipments in very small industries of the firm to be entirely intra-firm. This also prevents the spurious adding / dropping of products simply because of changes to the PT forms over the years.

Firm Trade Data. The LFTTD contains the value of all import and export transactions, by

[50]This process likely underestimates the significance of multi-product activity in the US economy, for two reasons. First, the long-form elicits questions about product sales over a pre-specified list of products (specific to the plant's classified industry). Although there is space for the firm to report shipments in products not covered by that pre-specified list, in practice firms rarely do. Second, the long-forms do not cover all single-establishment firms in the economy. A single-establishment firm could be selling in multiple industries but I will not see the breakdown of its sales over these industries if the firm was not sent a long-form.

trading country and by HS10 product, that each firm entity (a set of EIN tax codes) is a counterparty to. The CMF also contains data on plant-level shipments that are ultimately destined for export markets (whether directly or indirectly through an intermediary). If the plant is a multi-industry plant, I apportion this plant-level shipment across the plant's industries using product trailer product shipment shares. I use both LFTTD and CMF sources of data on exports to construct the export demand shock, detailed below. Data on firm exports and imports from Table 1 come from the LFTTD.

Country-level Trade Data. I use data from BACI and Comtrade (bilateral country-level trade flows at the HS6 level) to generate the five-year growth rates in imports of a destination n in product h used in the analysis, $\Delta \log IMP_{nht}$.

Knowledge Inputs. I use BEA input-output tables from 1997 for information on the use of inputs by industry. Table 7 lists the input industries from BEA input-output and capital flow tables that I classify as knowledge inputs. These correspond to NAICS sectors 55, 54, 51, and 533. Although results are robust to including finance, insurance, real estate, and other rental leasing (NAICS 52, 531, and 532), I do not include them in my list of knowledge inputs because of the separate way that financial inputs affect businesses compared to real inputs.[51]

The input-output tables record expenses on inputs whose value fully depreciates within one year (in the accounting sense). Because some intangible assets also have arbitrary depreciation rates and there are arbitrary rules around which inputs are expensed versus capitalized, I incorporate data from the capital flow tables on capitalized investments made by firms in manufacturing industries on knowledge input industries (for example, a shoemaker investing in software capital). I count both capitalized investments and expensed investments as knowledge input expenditures. The last three columns of Table 7 show aggregate expenditures on these input industries. I give statistics on the weighted mean (total manufacturing) expenditure share, as well as the 25th and 75th percentiles across industries.

In practice, it makes no difference to the results if I exclude capitalized expenditures from the capital flow tables. Most intangible inputs circa 1997 were still expensed under the national accounts. Only four input industries had capitalized investments: software publishers (511200), architectural, engineering, and related services (541300), custom computer programming services (541511), and computer systems design services (541512). Total capitalized investments in these industries by the manufacturing sector only amount to 0.64% of gross manufacturing output. I do not use input-output table data on knowledge input expenditures after 1997 because of subsequent changes to accounting rules that generate a lot of time variation in the data series.

Industry Definition. I construct a unified industry nomenclature, BEAX, that is time-invariant

[51] My model does not speak to the various mechanisms explored in the corporate finance literature, such as internal capital markets (Stein, 1997) or corporate socialism (Scharfstein and Stein, 2000). Instead, I soak up these effects under estimate properties of residual capital inputs in my quantitative framework, and under the $\Delta \log S_{fj}^{OTHER,SYM}$ control variable in the reduced-form.

Table 7: Definition of Knowledge Inputs and their Use in Manufacturing in 1997

		Share of Gross Output (%)		
$m \in \mathcal{M}^{KLG}$	Description	Mean	25th pctile	75th pctile
550000	Management of companies and enterprises	3.54	2.60	4.94
541700	Scientific research and development services	0.62	0.25	0.96
541300	Architectural, engineering, and related services†	0.62	0.31	0.96
5419A0	All other professional, scientific, and technical services	0.61	0.61	0.63
541511	Custom computer programming services†	0.58	0.21	0.86
541800	Advertising, public relations, and related services	0.48	0.14	0.62
541610	Management consulting services	0.28	0.28	0.30
541100	Legal services	0.28	0.09	0.30
541200	Accounting, tax prep., bookkeeping, & payroll services	0.15	0.08	0.21
541400	Specialized design services	0.09	0.01	0.02
541512	Computer systems design services†	0.07	0.02	0.06
54151A	Other computer related services	0.04	0.02	0.04
5416A0	Environmental and other technical consulting services	0.04	0.01	0.02
541940	Veterinary services	0.00	0.00	0.00
541920	Photographic services	0.00	0.00	0.00
533000	Lessors of nonfinancial intangible assets	0.69	0.06	0.34
5111A0	Wired telecommunications carriers	0.34	0.17	0.37
511200	Software publishers†	0.33	0.05	0.19
518200	Data processing, hosting, and related services	0.20	0.17	0.26
512100	Motion picture and video industries	0.03	0.00	0.03
512200	Sound recording industries	0.00	0.00	0.00
	Total Share of Expenditures on Knowledge Inputs	9.01	6.38	11.48

Notes: Mean refers to the weighted average across all 206 BEAX manufacturing industries, with industry gross output as weights. 25th and 75th pctile refers to expenditure shares of the corresponding percentiles (unweighted) across the 206 manufacturing industries. Codes in the first column refer to BEAX codes that are hand-developed; they roughly correspond to codes available in BEA I/O tables but are aggregated to ensure consistency over time.
Source: BEA Input-Output & Capital Flow Tables, 1997.
† Indicates industries where data on capitalized investments from the capital flow tables are used to compute expenditures. This makes up only 0.64% of gross manufacturing output.

over the period 1997 and 2012 and concordable with HS, NAICS, and BEA industry codes in each year. There are 206 BEAX industries in manufacturing. I use the HS-NAICS concordance in US Census Bureau data provided by Schott (2008) and Pierce and Schott (2012) to convert import and export HS codes (at the 10-digit and 6-digit levels) in each year to NAICS. I use the concordances provided by US Census Bureau and BEA to go between NAICS codes and BEA codes in each year. I use an iterative algorithm to aggregate over m:m splits over years and in each cross section so that in any given year, each NAICS code and HS10 code is entirely contained within a BEAX code.

A.2 The Non-Manufacturing Sector

Statistics shown in Table 1 are limited to the manufacturing sector since this paper limits attention to economies of scope in manufacturing. Here I briefly extend the reported statistics on multi-industry firm activity using a different classification of multi-industry firms: whether their sales span two or more industries in *any* sector. In the non-manufacturing sector, I find that 52% of sales come from multi-industry firms in 1997, while this number rises to 57% in 2012. The main difference compared to manufacturing is the long tail of small firms in non-manufacturing industries. The share of firms that are multi-industry in non-manufacturing is only 1% (compared to 20% in manufacturing). However, the lack of comparable product-trailer data for the non-manufacturing sector makes an apples-to-apples comparison difficult. In the non-manufacturing sector, industry variation in the data comes exclusively from the span of plants owned by a firm and the extent to which these plant classifications differ.

A.3 Export Demand Shocks

I leverage both the LFTTD and CMF sources of data on firm-industry exports to construct demand shocks, $\Delta \log S_{fjt}$. First, among LFTTD data, I compute export shares of each industry of each firm across destinations n and HS6 products h. I exclude destination-product markets whenever the firm's exports in those markets exceed 10% of the market's imports from the rest of the world. I use these shares as $s_{fnh|fj,t-1}$ in the analysis. Data on export intensity, $s^*_{fj,t-1}$, come from the CMF export shipment response variable. This is a firm-industry level variable as described in the preceding section. If the firm has no reported exports in an industry by manufacturing plants producing in that industry, it is likely that customs data is an instance of carry-along trade, made by the firm's wholesale / retail arm. These demand shocks are unlikely to affect the firm's manufacturing sales (reported by its plants) any more than other firms in the industry. Export intensity helps to discipline the customs-derived export demand shocks. I also set export intensity to zero for instances where carry-along trade of the firm (customs exports less census exports) in an industry exceeds its total external shipments in the CMF. After purging these edge cases, I am left with two measures of export intensity: (i) census exports divided by census sales in an industry, and (ii) customs exports divided by census sales in an industry. I take the average of these two

Table 8: Summary Statistics on Key Regression Sample

Statistics by firm-industry:	Variable	Mean	Std. Dev.
Sales Growth	$\Delta \log X_{fjt}$	0.15	0.99
Has Export Demand Shock?		0.68	0.47
Export Intensity	$s^*_{fj,t-1}$	0.06	0.10
Own-industry Demand Shock	$\Delta \log S_{fjt}$	0.028	0.082
Other-industry Shock, sales-weighted	$\Delta \log S_{fjt}^{OTHER,SYM}$	0.025	0.063
Other-Industry Shock, knowledge-expenditure-weighted	$\Delta \log S_{fjt}^{OTHER,KLG}$	0.002	0.007
Initial Period Sales (millions)	$X_{fj,t-1}$	165	1225
Initial Period Employment		522	2245

Other Statistics	Value
Number of firms in 1997-2002	5000
Number of firms in 2002-2007	4700
Share of U.S. manuf. sales accounted for by sample	0.51
Share of U.S. manuf. employment accounted for by sample	0.37

Notes: This table reports sample statistics for the particular sample of multi-industry firms and industries used in the reduced-form regression (Table 2). Number of observations are rounded for disclosure avoidance. The selection criteria is any firm-industry with continuing sales over a 5-year period, and belonging to a firm with at least one industry exporting (so that at least one out of the own-industry and cross-industry demand shock variables will be non-zero).

measures as my measure of $s^*_{fj,t-1}$.

A.4 Regression Analysis

A.4.1 Summary Statistics

Table 8 displays summary statistics on common variables that appear in regression Table 2 of the main text. The regression sample consists of all continuing firm-industries (across 5-year periods) of firms that have at least one industry with a non-zero export demand shock.

A.4.2 Export Demand Shock

Before proceeding to the main test of spillovers, I verify that demand shifters are indeed able to shift firm sales in the same industry by running the following regression for only the sample of firm-industries that have non-zero own-industry export demand shocks:

$$\Delta \log X_{fjt} = \alpha \Delta \log S_{fjt} + Controls_{jt}(s^*_{fj,t-1}) + FE_{jt} + \epsilon_{fjt},$$

where $Controls_{jt}(s^*_{fj,t-1})$ refers to various ways of controlling for the export intensity scaling variable, to make sure that variation in the export demand shock is not driven by firms with

Table 9: Relevance of Export Demand Shocks for Predicting Sales Growth

$\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)
$\Delta \log S_{fjt}$	0.59***	0.36***	0.34***	0.32***
	(0.10)	(0.12)	(0.11)	(0.11)
Industry-year-FE	✓	✓	✓	✓
$s^*_{fj,t-1}\times$ year-FE		✓	✓	
$s^*_{fj,t-1}\times$ Industry-year-FE				✓
Control for pre-period sales, $\log X_{fj,t-1}$			✓	✓
Observations	14,500	14,500	14,500	14,500
R^2	0.08	0.08	0.12	0.15

Notes: This table displays responses of firm-industry sales to demand shocks the firm receives in the same industry, in 5-year differences over the period 1997-2007. Standard errors in parentheses are clustered at the firm level. Number of observations are rounded for disclosure avoidance. Observations are at the firm-industry-year level, for continuing industries of all multi-industry firms that have an export demand shock in that same industry. The control $s^*_{fj,t-1}$ is the firm's export intensity (exports over sales) in industry j in the initial census year.

different export intensities being on different growth trends. Results are presented in Table 9. Across all three columns (that vary in terms of the control for export intensity used), the coefficient on the shock variable is positive and ranges from 0.32 to 0.59. Without controls for export intensity (column 1), the impact of the demand shock is statistically higher, consistent with selection on export intensity.[52]

A.4.3 Control Variables

The regression results in Table 2 are robust to including an exhaustive set of controls, as described in the main text. I define the pre-period size of the firm as the log of the sum of sales over all the firm's industries. The pre-period industry size is the log of the firm's sales in that industry. I define export status as a dummy variable equal to 1 if the firm has a non-zero export intensity in that industry, $s^*_{jt} > 0$.

In column (1) of Table 10, I re-estimate specification (5) of Table 2 but including all the controls discussed in the body of the text. I find that—aside from an increase in R^2—there are no changes to the magnitude or significance of spillover coefficients.

[52] I also run a placebo test where I assign firms in a given industry j a random ΔSHK_{fjt} drawn from the empirical distribution of shocks received by all firms active in industry j. The placebo tests return false positives in column (1) but not columns (2) and (3). This suggests that linear controls for export intensity control adequately for selection on export intensity.

Table 10: Cross-Industry Spillovers within the Firm: Additional Robustness Specifications

Sales growth, $\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)
Own-industry shock	0.51***		0.47***	0.47***
$\Delta \log S_{fjt}$	(0.10)		(0.09)	(0.09)
Cross-industry shocks				
(i) Knowledge-based spillovers	8.26***	6.54***	7.02***	8.14***
$\Delta \log S_{fjt}^{OTHER,KLG}$	(2.22)	(2.08)	(2.11)	(2.22)
(ii) Symmetric spillovers	-0.81***			-0.48
$\Delta \log S_{fjt}^{OTHER,SYM}$	(0.26)			(0.39)
(iii) Physical-input spillovers		-0.75***	-0.86***	-0.44
$\Delta \log S_{fjt}^{OTHER,PHY}$		(0.26)	(0.26)	(0.44)
Industry-year-FE	✓	✓	✓	✓
Full Set of Controls	✓			
Observations	21,500	21,500	21,500	21,500
R^2	0.12	0.05	0.06	0.06

Notes: This table displays additional specifications using the same sample of firms as regression Table 2. Standard errors are clustered at the firm level. Number of observations rounded for disclosure avoidance. Observations are at the firm-industry-year level, for continuing observations of a sample of multi-industry firms with at least one industry directly exporting. The full set of control variables include: initial period firm size, firm-industry size, export status, export intensity, as well as controls for the shares in the functional forms (SYM and KLG) used to collapse shocks in other industries, and the interaction of these shares with other initial-period firm-industry variables.

A.4.4 Other Input Linkages

The remaining columns (2)-(4) of Table 10 estimates a variant of the main regression equation (1) where instead of using sales weights to form $\Delta \log S_{fjt}^{OTHER,SYM}$, I focus only on input proximity and separate out knowledge inputs from the remaining inputs in the BEA I/O tables. I call the remaining set of inputs physical, denoted by PHY, and construct $\Delta \log S_{fjt}^{OTHER,PHY}$ in the same way as $\Delta \log S_{fjt}^{OTHER,KLG}$. Column (2) shows that they pull in opposite directions within the firm, strongly suggestive that industry spillovers differ in the knowledge input dimension. Column (3) adds the own-industry shock to the regression, and column (4) includes both $\Delta \log S_{fjt}^{OTHER,PHY}$ and $\Delta \log S_{fjt}^{OTHER,SYM}$.

To understand whether there are any discernible inputs under the PHY input category that drives negative spillovers, I conduct a placebo exercise based on column (5) of regression Table 10 where I use different proximity measures constructed based on bilateral similarity in use over different sets of inputs (in lieu of knowledge inputs, KLG). Table 11 displays the regression table counterpart to coefficients shown in Figure 2. The numbers next to the description in parentheses display the BEAX subroot (1, 2 or 3 digits) among 6-digit BEAX industry inputs that make up the sector. Taxes, government sector inputs, and the two types of value-added are specific BEA categories that have no corresponding numeric BEAX code.

A.4.5 Firm-level Respones

I construct weights, $\eta_{fk,t}$, in the firm-level regression equation (3) for the various outcome categories as follows:

$$\eta_{fkt} = \frac{\beta_{k,y} X_{fkt}}{\sum_{k'} \beta_{k',y} X_{fk't}},$$

where X_{fkt} is firm sales in industry k and $\beta_{k,y}$ takes on the following values depending on the outcome variable y:

(i) Purchased professional services: $\beta_{k,y} = \beta_{k,INT}$, the share of gross output by industry k on knowledge inputs.

(ii) Sales: $\beta_{k,y} = 1$ (so η are simply sales shares).

(iii) Capex: $\beta_{k,y} = \beta_{k,CAP}$, the share of gross output by industry k on capital value added.

(iv) Payroll: $\beta_{k,y} = \beta_{k,LAB}$, the share of gross output by industry k on labor value added.

Data on purchased professional services at the firm level come from aggregating responses of plants of the firm to the following ASM survey expense line items: expenses on legal, accounting, management, communication, advertising, and computer software and data processing services. Firms that do not have plants respond to these questions in the ASM and firms that have sales outside of the manufacturing sector account for larger than 5% of overall sales are dropped for this particular regression. Data on firm-wide capex come from summing up plant-level capital expenditures, and data on payroll come from summing up plant-level production worker payroll.

A.4.6 Threats to Identification

Related to the discussion on threats to identification in Section 1.4, I directly test and reject the hypothesis that the import growth patterns across industries within a destination are positively correlated among knowledge-input intensive industries. I aggregate imports of each destination to the industry level (across products), $IMP^{US}_{nk,t-1}$, and construct spillover functions corresponding to the same functional forms used in the main firm-industry regression table:

$$\Delta \log IMP^{\sim US,OTHER,BLK}_{njt} \equiv \sum_{k \neq j} \sum_{m \in \mathcal{M}^{BLK}} \beta_{jm} \left(\frac{\beta_{km} IMP^{\sim US}_{nk,t-1}}{\sum_{k \neq j} \beta_{km} IMP^{\sim US}_{nk,t-1}} \right) \Delta \log IMP^{\sim US}_{nkt},$$

for $BLK = \{KLG, PHY\}$ to separate out knowledge inputs from remaining (physical) inputs in the input-output tables. I then run the following regression, at the level of destination-industries, over the same time period (in 5-year differences):

$$\Delta \log IMP^{\sim US}_{njt} = \psi^{PHY} \Delta \log IMP^{\sim US,OTHER,PHY}_{njt} + \psi^{KLG} \Delta \log IMP^{\sim US,OTHER,KLG}_{njt} + FE_{jt} + FE_{nt}.$$

I do not find that ψ^{INT} is positive, either with or without destination-year fixed effects.

A.4.7 Vertical Explanations

There are four general reasons a demand shock in industry k may propagate within a firm to generate increased sales in industry j: (i) j supplies k, (ii) k supplies j, (iii) k, j use similar inputs, and (iv) k, j are demand-complementary and have similar buyers. My focus is on mechanism (iii). The discussion in the body of the paper (Section 1.4) rules out reason (iv), demand-complementarity. The following points discuss the first two, vertical mechanisms.

(i) I use the external sales growth in industry j as the main sales outcome variable, so the regressions pick up the firm's change in external sales in industry j. It could still be true that external sales growth is driven by productivity effects induced by intra-firm sales growth. I check specifically whether intra-firm sales growth in j occurs in response to demand shocks in k. I find that they do not respond (even among only the tiny fraction of j industries that have any inter-plant shipments at all).

(ii) For this to occur there must first be an increase in internal shipments in the shocked industry k. Then the story would be that increased quality of shipments (as measured by increased internal sales) drives productivity growth in industry j. I use growth in inter-plant (intra-firm) shipments as an outcome variable across the specifications in Table 9. I find that they do not respond (even among the tiny fraction of k industries that have any inter-plant shipments at all).

A.4.8 Deflating

Even though the main regressions specifications all include industry-year fixed effects, whether variables are nominal or deflated (with industry-deflators) could make a difference in terms of the relative sizes of export shares and expenditure shares. All the reduced-form results are virtually unchanged when the following variables are deflated with industry-level price deflators from the NBER-CES manufacturing database: demand shocks (import growth at destinations), outcomes (external shipments of a firm-industry), as well as 'initial-period' variables in the formulation of weights, for example, behind the $\Delta \log S_{fjt}^{OTHER}$ functions.

Table 11: Intensive Margin Cross Industry Spillovers: Placebo Input Linkages

Sales growth, $\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Own-industry shock	0.48***	0.45***	0.46***	0.46***	0.46***	0.45***	0.45***	0.45***	0.45***	0.46***	0.45***	0.45***	0.45***	0.45***
Sales-weighted linkage	-0.26**	-0.68**	-0.49**	-0.82*	-0.18	-0.45*	-0.12	-0.15	-0.29	-0.00	-0.07	0.25	-0.05	0.19
$h_{fj}(.)$ based on the following input linkages														
Leasing of Intangibles (533)	27.73***													
Headquarter Services (55)		15.82**												
Professional Services and Information (54, 51)			8.262*											
Finance, Insurance, and Real Estate (52, 531)				40.26										
Leasing of Tangibles (532)					24.92									
Transportation, Wholesale, and Retail (4)						14.94*								
Taxes and Government							3.603							
Utilities and Construction (2)								3.413						
Capital Value-Added (Gross Operating Surplus)									1.766					
Labor Value-Added										-0.325				
Agriculture (1)											-0.715			
Manufacturing (3)												-0.889		
Administrative Services (56)													-4.305	
All Other Services (6, 7, 8, 9)														-20.8
Industry-year-FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500
R^2	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06

Notes: This Table reproduces the benchmark specification in column (5) of Table 2 but with alternative other-industry demand shocks (given by the rows of the table). Standard errors clustered at the firm level. Observations at the firm-industry-year level, for continuing observations of a sample of multi-industry firms with at least one industry directly exporting.

B Theory Appendix

In this section I prove any claims and derive the expressions found in Propositions and Lemmas in the main body of the paper. I also provide commentary on a few relevant extensions of the results of the model.

B.1 Proof of Lemma 1: Option Value from Mutability and Scope

Combine the expression in Assumption 2 with the expression for firm-industry profits in equation (5) to compute the expected impact on firm gross profits from the arrival of a piece of capital i of type m:

$$\frac{\Delta_{fm}}{Z} \equiv \mathbb{E}\left[\max_j \tilde{\alpha}_{mj} B_j \xi_{fj}\, \phi_{fjm,i}\right],$$

where $\phi_{fjm,i}$ is an independent random draw from a Fréchet distribution. The expected impact is the change in gross profits in the industry in which the capital (conditional on the match-specific draws of $\phi_{fjm,i}$ in different industries) generates the highest improvement in profitability. The remainder of this proof simply relies on properties of the Fréchet distribution popularized by Eaton and Kortum (2002). I can re-express the profit contribution as:

$$\frac{\Delta_{fm}}{Z} = \mathbb{E}\left[\max_j \tilde{\phi}_{fjm,i}\right],$$

where $\tilde{\phi}_{fjm,i}$ is an independent random draw from a different Fréchet distribution that absorbs the multiplicative shifters:

$$Pr(\tilde{\phi}_{fjm,i} \leq x) = e^{-\left(\tilde{\alpha}_{mj} B_j \xi_{fj}\right)^{\theta_m} x^{-\theta_m}}, \qquad \forall j \in \mathcal{J},$$

and it follows that

$$\frac{\Delta_{fm}}{Z} = \left(\sum_j \left(\tilde{\alpha}_{mj} B_j \xi_{fj}\right)^{\theta_m}\right)^{\frac{1}{\theta_m}} \Gamma(1 - 1/\theta_m),$$

$$\Delta_{fm} = \left(\sum_j \delta_{fmj}^{\theta_m}\right)^{\frac{1}{\theta_m}}.$$

where Γ is the gamma function and $\delta_{fmj} \equiv \xi_{fj} \alpha_{mj} B_j Z$.

B.2 Proof of Lemma 2: The Firm's Solution

In stage II, the firm decides its expenditures on production inputs given its accumulated capital, $\{\varphi_{fj}\}_{j\in\mathcal{J}}$. This problem is separable by industry. The solution to this under monopolistic competition yields the expression for gross profits of the firm (sales less production input expenses) in equation (5).

At the beginning of stage I, the firm decides its expenditures on capital inputs. Throughout stage I, the firm deploys each new capital opportunity to a chosen industry. Given the memorylessness of capital deployment induced by Assumption 2, expected firm net profits Π_f can be written as:

$$\mathbb{E}[\Pi_f] = \max_{\{\iota_{fm}\}_{m\in\mathcal{M}}} \mathbb{E}\left[\sum_j \sum_m B_j \xi_{fj} \tilde{\alpha}_{mj} \sum_i^{A_{fm}} \phi_{fjm,i} \mathbf{1}_{fjm,i}\right] - \sum_m w\iota_{fm}.$$

The first half of the expression denotes the expected gross profits of the firm given its how its choices of capital inputs ι_{fm} affect the Poisson arrival rate of capital A_{fm} and its deployment decisions $\mathbf{1}_{fjm,i}$. The second half of the expression relates to the costs of capital inputs. I assume that the firm can purchase capital inputs at constant unit costs w. (Any differences in unit prices across types of capital inputs are absorbed by technology shifters α_{fm}). Given the independence of the Poisson and Fréchet distributions, this problem is additively separable, so that the deployment of each capital that arrives is independent of past and future decisions.

More formally, the deployment decision $\mathbf{1}_{fjm,i}$ has the following expectational properties inherited from Fréchet (Section B.1):

$$\begin{aligned} Pr(\mathbf{1}_{fjm,i} = 1) &= Pr(j = \arg\max_{k\in\mathcal{J}} \tilde{\phi}_{fkm,i}) \\ &= \frac{\delta_{fmj}^{\theta_m}}{\Delta_{fm}^{\theta_m}} \equiv \mu_{fmj}, \end{aligned}$$

where μ_{fmj} are deployment probabilities for any given arrival of capital of type m and

$$\mathbb{E}[\tilde{\alpha}_{mj} B_j \xi_{fj} \phi_{fjm,i} | \mathbf{1}_{fjm,i} = 1] = \mathbb{E}\left[\max_j \tilde{\alpha}_{mj} B_j \xi_{fj}\, \phi_{fjm,i}\right] = \frac{\Delta_{fm}}{Z},$$

by Lemma 1. Recalling that A_{fm} is distributed independently with Poisson mean $Z\left(\frac{\rho_m}{\rho_m-1}\iota_{fm}\right)^{\frac{\rho_m-1}{\rho_m}}$, expected firm net profits Π_f can be re-written as

$$\begin{aligned} \mathbb{E}[\Pi_f] &= \max_{\{\iota_{fm}\}_{m\in\mathcal{M}}} \sum_m \sum_j \mathbb{E}[A_{fm}|\iota_{fm}] \mathbb{E}[\tilde{\alpha}_{mj} B_j \xi_{fj} \phi_{fjm,i} | \mathbf{1}_{fjm,i} = 1] Pr(\mathbf{1}_{fjm,i} = 1) - \sum_m w\iota_{fm}. \\ &= \max_{\{\iota_{fm}\}_{m\in\mathcal{M}}} \sum_j \sum_m \left(\frac{\rho_m}{\rho_m - 1}\iota_{fm}\right)^{\frac{\rho_m-1}{\rho_m}} \Delta_{fm}\mu_{fmj} - \sum_m w\iota_{fm}. \\ &= \max_{\{\iota_{fm}\}_{m\in\mathcal{M}}} \sum_m \left(\frac{\rho_m}{\rho_m - 1}\iota_{fm}\right)^{\frac{\rho_m-1}{\rho_m}} \Delta_{fm} - \sum_m w\iota_{fm}.. \end{aligned}$$

This is a simple convex optimization problem separable across capital input types m, with solution given by

$$\iota_{fm} = \frac{\rho_m - 1}{\rho_m} \Delta_{fm}^{\rho_m} w^{-\rho_m}, \qquad \forall m,$$

and thus net profits are equal to

$$\begin{aligned} \mathbb{E}[\Pi_f] &= \sum_m \Delta_{fm}^{\rho_m} w^{1-\rho_m} - \sum_m \frac{\rho_m - 1}{\rho_m} \Delta_{fm}^{\rho_m} w^{1-\rho_m} \\ &= \sum_m \frac{1}{\rho_m} \Delta_{fm}^{\rho_m} w^{1-\rho_m}. \end{aligned}$$

Likewise, expected gross profits in a single industry j are given by

$$\mathbb{E}[\pi_{fj}] = \sum_m \mu_{fmj} \Delta_{fm}^{\rho_m} w^{1-\rho_m}.$$

The probability that a firm is active in industry j, denoted $\chi_{fj} = 1$, is one minus the probability that no capital is deployed to that industry. Since deployment of capital is independent across opportunities, and the total arrival rate of capital of any type m is a Poisson process with rate A_{fm}, the arrival of ideas *deployed to industry* j is also a Poisson process, with different rate $A_{fm}\mu_{fmj}$. The probability that no capital is deployed is thus the probability that there are no arrivals from the joint Poisson processes of deployed capital to industry j over all capital types:

$$Pr(\chi_{fj} = 1) = 1 - \exp\left(\sum_m \mu_{fmj} A_{fm}\right) = 1 - \exp\left(-Z \sum_m \delta_{fmj}^{\theta_m} \Delta_{fm}^{\rho_m - 1 - \theta_m} w^{1-\rho_m}\right),$$

and is independent across industries. Similarly, an inactive firm is a firm with no capital arrive at all. The probability that a firm is inactive is thus given by

$$Pr(\chi_f = 1) = 1 - \exp\left(\sum_m A_{fm}\right) = 1 - \exp\left(-Z \sum_m \Delta_{fm}^{\rho_m - 1} w^{1-\rho_m}\right).$$

B.3 Proof of Proposition 1: Spillovers within the Firm

Log-differentiating equation (8) with respect to shifters of firm profitability in industries k, holding factor prices w constant, yields

$$\begin{aligned} \mathrm{d}\log \mathbb{E}[X_{fj}] &= \mathrm{d}\log \mathbb{E}[\pi_{fj}] \\ &= \sum_m \lambda_{fjm} \left(\theta_m \mathbf{1}_{k=j} \, \mathrm{d}\log \xi_{fk} B_k + (\rho_m - \theta_m) \sum_k \mu_{fmk} \, \mathrm{d}\log \xi_{fk} B_k\right), \end{aligned}$$

where μ_{fmj} are capital deployment shares given in Lemma 2, and λ_{fjm} denote utilization shares: the share of gross profits of industry j attributable to capital contributions of type m:

$$\lambda_{fjm} \equiv \frac{\mu_{fmj}\Delta_{fm}^{\rho_m} w^{1-\rho_m}}{\sum_{m'} \mu_{fm'j}\Delta_{fm'}^{\rho_{m'}} w^{1-\rho_{m'}}}.$$

B.4 Connecting Firm-level Elasticities in the Model and Reduced-Form

The firm-level cross-industry elasticity from Proposition 1 combines responses on both intensive and extensive margins ($\mathbb{E}[X_{fj}]$ includes the non-trivial probability of zero sales).[53] But for sufficiently large firms (high in ξ_f), all the responses load on the intensive margin. The intuition is that the largest firms choose a level of capital expenditures so high to start with that the likelihood of cross-industry shocks affecting the extensive margin vanishes. For example, a demand shock for General Electric's MRI machines might affect GE's intensive margin sales of jet engines but is unlikely to affect whether the company is active at all in the jet engine business. With a high enough arrival rate, the expectation operator becomes exact thanks to the law of large numbers.[54] The following Lemma clarifies this point and motivates the focus of the reduced-form regressions on the intensive margin (given that the regression sample comprises large firms):

Lemma 3 (Intensive Margin Spillovers for Large Firms) *For a firm with non-zero latent productivities ξ_{fj} and ξ_{fk}, cross-industry elasticities characterized by Proposition 1 load completely onto the intensive margin as the average firm productivity ξ_f become arbitrarily high:*

$$\lim_{\xi_f \to \infty} \frac{\mathrm{d}\log \mathbb{E}[X_{fj}]}{\mathrm{d}\log \xi_{fk} B_k} = \frac{\mathrm{d}\log \mathbb{E}[X_{fj}|X_{fj} > 0]}{\mathrm{d}\log \xi_{fk} B_k}.$$

As a corollary, the share of the cross-industry elasticity in Proposition 1 explained by extensive margin changes within the firm ranges from 1 (for the lowest ξ firms) to 0 (for the highest ξ firms).

Proof. Separate the expected gross sales into intensive margin and extensive margins:

$$\log \mathbb{E}[X_{fj}] = \log \mathbb{E}[X_{fj}|X_{fj} > 0] + \log Pr(X_{fj} > 0).$$

Differentiate the extensive margin:

$$\frac{\mathrm{d}\log Pr(X_{fj} > 0)}{\mathrm{d}\log \xi_{fk} B_k} = \frac{\exp(-\Sigma_{fj})\Sigma_{fj}}{1-\exp(-\Sigma_{fj})} \sum_m s_{mj} \left(\theta_m \mathbf{1}_{k=j} + (\rho_m - \theta_m)\mu_{fmk}\right),$$

[53] It is easy to log-differentiate equation (9) to derive purely extensive margin predictions and thus decompose the action.

[54] All the Frechet idiosyncratic errors wash out. This large firm limit also corresponds to the framework pioneered in Tintelnot (2016) and Antràs et al. (2017), whereby outcomes are smoothed across a continuum within the firm instead of being granular.

where

$$s_{mj} \equiv \frac{Z\mu_{fmj}\Delta_{fm}^{\rho_m-1}w^{1-\rho_m}}{\Sigma_{fj}},$$

and $\Sigma_{fj} \equiv Z\sum_m \mu_{fmj}\Delta_{fm}^{\rho_m-1}w^{1-\rho_m}$. Let $\bar{\xi}_f$ denote the average of the profitability shifters in two industries of the firm j,k in which the firm has non-zero sales. The term to the right of the $\sum_m$ in the derivative of the extensive margin is bounded (weighted average of elasticities),

$$\lim_{\bar{\xi}_f\to\infty} \frac{\mathrm{d}\log Pr(X_{fj}>0)}{\mathrm{d}\log \xi_{fk}B_k} = \lim_{\Sigma_{fj}\to\infty} \frac{\exp(-\Sigma_{fj})\Sigma_{fj}}{1-\exp(-\Sigma_{fj})} = 0,$$

where the last equality makes use of L'hopital's rule. ■

C Quantitative Appendix

C.1 Identification of Macro Variables

Conditional on micro parameters Θ, γ, I use the aggregate predictions of the parametrized model to invert for macro variables: technology coefficients α_{mj}, residual profits in each year, B_{jt}, and the technology index in the cost function, Z_t.

First, to identify α_{mj} from base period ($t = 1$) data on M_{mj}, I solve the second line of equation (12) separately for each of the three types of knowledge capital types $m \in \mathcal{M}^{KLG}$. For each type of knowledge input m, I invert a separate J-system of equations for a J vector of model variables $\alpha_{mj}B_{j,t=1}Z_t$ (grouped together) given data on capital expenditures of that type, $\{M_{mj}\}_j$.

The mean of α_{mj} across m for each j is isomorphic to a constant term in $B_{j,t=1}$. Thus, I am free to normalize the technology coefficient of the last capital input type $\alpha_{RES,j} = 1$. To find base period $B_{j,t=1}$, I subtract knowledge expenditures (the second line of equation 12 for all $m \in \mathcal{M}^{KLG}$) from gross profits (the first line of equation 12) to yield

$$\frac{\pi_j - \sum_{m\in\mathcal{M}^{KLG}} \frac{\rho^{KLG}}{\rho^{KLG}-1}M_{mj}}{N} = \int_\xi \delta_{f,RES,j}^{\theta^{RES}}\Delta_{f,RES}^{\rho^{RES}-\theta^{RES}}\,\mathrm{d}G(\xi),$$

where the left-hand-side is data contained in BEA input-output tables, and the right-hand side contains a J vector of unknowns $B_{j,t=1}Z$ (since $\alpha_{RES,j} = 1$) to be inverted.[55] Note that this equation imposes a non-negativity restriction which manifests as a lower bound on the value of ρ^{KLG}

[55]Note that no expenditure data is needed on the residual capital input category. The residual category is set up to also absorb payments to latent factors (venture capital, sweat equity).

according to the model:

$$\pi_j > \sum_m \frac{\rho^{KLG}}{\rho^{KLG}-1} M_{mj}, \qquad \forall j$$
$$\iff (1-\varsigma_j) > \frac{\rho^{KLG}}{\rho^{KLG}-1} \sum_{m \in KLG} \beta_{mj}, \qquad \forall j$$
$$\iff \frac{\rho^{KLG}-1}{\rho^{KLG}} > \max_j \frac{\beta_{KLG,j}}{1-\varsigma_j} = \max_j \frac{\beta_{KLG,j}}{\beta_{KLG,j}+\beta_{RES,j}},$$

for BEA expenditure shares β_{mj}. In the data, this restriction corresponds roughly to imposing that $\rho^{KLG} > 3$.

I use the values of $B_{j,t=1}Z_{t=1}$, and $\alpha_{mj}B_{j,t=1}Z_{t=1}$ to back out α_{mj}. I hold α_{mj} constant over all three time periods, for lack of input-output expenditure data on knowledge inputs in subsequent years.

To find future-period residual profits $\{B_{j,2}, B_{j,3}\}_j$ (jointly with Z_t), I invert for $B_{jt}Z_t$ using the first line of equation (12) with gross output data from that corresponding year and values of α_{mj} from the above two steps.

Finally, given the full set of $\{B_{jt}Z_t\}_{t=1,2,3}$ and $\boldsymbol{\alpha}$, I can solve for Z_t such that the share of single-industry firms in the model matches 0.8 in the data:

$$\frac{\int_\xi \sum_j \left(Pr(\chi_{fjt}=1) \prod_{k \neq j} (1 - Pr(\chi_{fkt}=1)) \right) \mathrm{d}G(\xi)}{\int_\xi Pr(\chi_{ft}=1) \, \mathrm{d}G(\xi)} = 0.8. \tag{14}$$

where the probabilities of entry by industry (χ_{fjt}) and by firm (χ_{ft}) are given in Section B.2.

C.2 Proof of Proposition 2: Inference of Scalability and Mutability

First, I show that at true parameter values $\Theta, \boldsymbol{\gamma}$, the following $J \times J$ structural moment conditions hold true:

$$\mathbf{E}_f \left[(\epsilon_{fjt} - \mathring{\epsilon}_{fjt-1}) \, \Delta \widetilde{\log S}_{fkt} \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right] = 0, \qquad \forall j, k, \forall t = \{2, 3\}$$

where $\Delta \widetilde{\log S}_{fkt}$ is the de-meaned shock among shocks received by all firms that are active in industry k, and $\mathring{\epsilon}_{fj,t-1}$ is the structural error conditional on the firm being active in industry j:

$$\mathring{\epsilon}_{fj,t-1} \equiv X_{fj,t-1} - \mathbb{E}[X_{fj,t-1} | \xi_{f,t-1}, \chi_{fj,t-1} = 1].$$

By the law of iterated expectations, the moment condition for any pair of industries j, k in any year $t = \{2, 3\}$ can be written as

$$\mathbf{E}_{\xi_{f,t-1}, \Delta \log S_{ft}} \left[\Delta \widetilde{\log S}_{fkt} \; \mathbf{E}_f \left[(\epsilon_{fjt} - \mathring{\epsilon}_{fjt-1}) \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1, \xi_{f,t-1}, \Delta \log S_{ft} \right] \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right],$$

where the $(\epsilon_{fjt} - \mathring{\epsilon}_{fjt-1})$ terms inside the inner expectation are zero in expectation because

$$\mathbf{E}_f \left[X_{fjt} \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1, \xi_{f,t-1}, \Delta \log S_{ft} \right] = \mathbb{E} \left[X_{fjt} \mid \xi_{f,t} \right],$$

using Assumption 4 (relevance), so that $\xi_{f,t}$ can be expressed as a known function of $(\xi_{f,t-1}, \Delta \log S_{ft})$), and

$$\mathbf{E}_f \left[X_{fj,t-1} \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1, \xi_{f,t-1}, \Delta \log S_{ft} \right] = \mathbb{E} \left[X_{fj,t-1} \mid \xi_{f,t-1}, \chi_{fj,t-1} = 1 \right],$$

using Assumption 5 (conditional independence), so that $\Delta \log S_{ft}$ can be dropped conditional on the industry presence χ and unobserved profitability shifters. Note that it is important for $\Delta \log S_{ft}$ to be independent of realized outcomes $X_{fj,t-1}$, so what Assumption 5 rules out is for firms that do unexpectedly well (conditional on ξ) to receive higher demand shocks.

Next, I show that inference can proceed off of computable sample analogs of the moment conditions. One component of the moment condition is pure data (the terms inside structural residuals ϵ that correspond to realized sales X_{ft}). I label the set of firms that are active in any pair of industries j, k in year $t-1$ by $n_{fj,t-1}$ and construct

$$\Xi^o_{jkt} \equiv \frac{1}{n_{jk,t-1}} \sum_{f \in n_{jk,t-1}} (X_{fjt} - X_{fj,t-1}) \, \Delta \widetilde{\log S}_{fkt}, \quad \forall j, k, \forall t = \{2, 3\}.$$

The remaining components of the structural residuals in the moment condition is model-implied sales (i.e. $\mathbb{E}[X_{fj,t-1} | \xi_{f,t-1}, \chi_{fj,t-1} = 1]$). I re-write this as:

$$\begin{aligned}
\Xi^m_{jkt} &\equiv \mathbf{E}_f \left[\left(\mathbb{E}[X_{fjt} | \xi_{ft}] - \mathbb{E}[X_{fj,t-1} | \xi_{f,t-1}, \chi_{fj,t-1} = 1] \right) \, \Delta \widetilde{\log S}_{fkt} \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right] \\
&= \mathbf{E}_{\xi_{f,t-1}, \Delta \log S_{ft}} \left[g_{jk}(\xi_{fj,t-1}, \Delta \log S_{ft}) \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right] \\
&= \mathbf{E}_{\Delta \log S_{ft}, \chi_{f,t-1}} \left[\int_{\xi_{f,t-1}} g_{jk}(\xi_{fj,t-1}, \Delta \log S_{fkt}) \; Pr(\xi_{f,t-1} \mid \chi_{f,t-1}) \, \mathrm{d}\xi_{f,t-1} \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right] \\
&= \mathbf{E}_{\Delta \log S_{ft}, \chi_{f,t-1}} \left[\int_{\xi} g_{jk}(\xi, \Delta \log S_{fkt}) \; \frac{Pr(\chi_{f,t-1} \mid \xi)}{\int_{\xi} Pr(\chi_{f,t-1} \mid \xi) \, \mathrm{d}G(\xi)} \mathrm{d}G(\xi) \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right]
\end{aligned}$$

where the second line uses the law of iterated expectations and introduces a g_{jk} function to capture the inner expectation terms:

$$g_{jk}(\xi_{fj,t-1}, \Delta \log S_{ft}) \equiv$$
$$\Delta \widetilde{\log S}_{fkt} \mathbf{E}_f \left[\left(\mathbb{E}[X_{fjt} | \xi_{ft}] - \mathbb{E}[X_{fj,t-1} | \xi_{f,t-1}, \chi_{fj,t-1} = 1] \right) \ |\Delta \log S_{ft}, \xi_{f,t-1}, \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right],$$

which is an analytical function of $(\xi_{fj,t-1}, \Delta \log S_{ft})$ given the solution properties of the model and again, given Assumption 4 (relevance), so that $\xi_{f,t}$ is known. The third line applies Assumption 5 (conditional independence) to get $Pr(\xi_{f,t-1} \mid \Delta SHK_{ft}, \chi_{f,t-1}) = Pr(\xi_{f,t-1} \mid \chi_{f,t-1})$ and the forth line applies Bayes rule to transform $Pr(\xi_{f,t-1} \mid \chi_{f,t-1})$ into known analytical extensive margin probabilities given the model.

The sample analog of the last line is given by

$$\Xi^m_{jkt} \equiv \frac{1}{n_{jk,t-1}} \sum_{f \in n_{jk,t-1}} \sum_{s \in S} \omega_{fs,t-1} \, m_{fjk}(\xi_s, \Delta \log S_{ft}) \, \Delta \widetilde{\log S}_{fkt},$$

where ω_{sf} are probability weights that stand for the probability that a simulated firm s has fundamentals ξ that belong to firm f in the data (with industry presence $\chi_{f,t-1}$) relative to other simulated firms $s' \in S$,

$$\omega_{fs,t-1} \equiv \frac{\prod_j Pr(\chi_{j,t-1} = \chi_{fj,t-1} | \xi_s)}{\sum_{s'} \prod_j Pr(\chi_{j,t-1} = \chi_{fj,t-1} | \xi_{s'})},$$

By the law of large numbers, $m_{jkt} \equiv \Xi^o_{jkt} - \Xi^m_{jkt}$ in the Proposition can be written as

$$\lim_{n_{jk,t-1} \to \infty} \lim_{s \to \infty} \Xi^o_{jkt} - \Xi^m_{jkt} = \mathbf{E}_f \left[(\epsilon_{fjt} - \mathring{\epsilon}_{fjt-1}) \, \Delta \widetilde{\log S}_{fkt} \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right] = 0.$$

C.3 External Validity

Table 12 displays the distribution of firms and sales over firm scope behind Figure 4. These are computed by simulating the actual outcomes of firms in the model when matched to data on output and input expenditure by industry in 1997 (which is done for estimation).

C.4 General Equilibrium Definition

I introduce some more notation used to characterize the open economy equilibrium. Take the perspective of US as the domestic economy trading with a set $\mathcal{D}^F$ of foreign countries. Let D denote the US manufacturing trade deficit (exports of non-manufacturing services) vis-à-vis the rest of the world. Let $\bar{Y}_{d,j}$ denote the total market size faced by US firms in each industry j in a foreign destination $d \in \mathcal{D}^F$, and suppose that all firms are common exporters.[56] Let $\bar{PX}_{dj}$

[56] My paper does not consider the selection-into-exporting margin. But in fact, the common-exporter assumption is not extreme. It suffices that firms have common ex-ante *expectations* of exporting. One

Table 12: Scope Distribution in the Data and Model, 1997

	Share of Firms (%)		Share of Sales (%)	
Number of Industries	Data	Model	Data	Model
1	80.99	80.15	26.13	25.00
2	13.01	13.54	10.80	15.74
3	3.32	3.29	7.34	9.43
4	1.33	1.07	5.51	6.33
5	0.61	0.44	4.08	5.48
6	0.28	0.23	2.89	5.76
7	0.14	0.15	3.54	6.71
8	0.08	0.10	3.63	4.05
9 +	0.25	1.03	36.05	21.49

Notes: The distribution of firms by scope, in the data and in the model (with the six estimated parameters, $\Theta, \gamma_0, \gamma_1$). Sales of firms with 9 or more industries could not be simulated via brute force due to memory issues when simulating the discrete Poisson process. Instead, it is backed out from the fact that the share of sales by firms with one industry was set to equal 0.25 in the estimation.

represent indices of price competitiveness in foreign market d by all non-US firms. Let $\bar{PM}_{dj}$ represent indices of price competitiveness in the US market by foreign firms from d. For example, an increase in $\bar{PM}_{china,j}$ indicates that prices of Chinese goods in the US have been lowered (become more competitive).

Definition 1 (General Equilibrium) *Given exogenous foreign price competitiveness abroad and at home, $\{\bar{PX}_{dj}, \bar{PM}_{dj}\}_{j\in\mathcal{J},d\in\mathcal{D}^F}$, foreign expenditures $\{\bar{Y}_{dj}\}_{j\in\mathcal{J},d\in\mathcal{D}^F}$, general equilibrium is described by a wage, w, and a vector of domestic price competitiveness:*

$$PD_j \equiv N \int_{\xi} \mathbb{E}\left[p_{fj}^{1-\sigma_j}\right] \mathrm{d}G(\xi) \quad \forall j \in \mathcal{J}$$

such that the following equations and related definitions hold:

(i) Total industry expenditures in the US is given by

$$Y_j = \sum_k \beta_{kj} X_k + \beta_{F,j}(wL),$$

where X_j stands for domestic industry gross output, β_{kj} is the share of gross output of industry

micro-foundation, for example, would be if each capital allocated to industry j has a probability of being used at the same time for export market production. A firm then enters into exporting if and only if it has a non-zero amount of capital adaptable for export markets. Despite this common probability of exporting, empirically larger firms would be more likely to export because of a higher chance of having at least some capital be adapted for export markets.

k expensed on inputs from industry j, and $\beta_{F,j}$ reflects the share of final consumption spent on manufacturing industry j.

(ii) Goods market clearing yields a system of J equations in J residual profit shifters B_j: output produced over all firms has to equal total domestic industry output, which has to equal output consumed at home plus output exported to foreign markets:

$$\begin{aligned} X_j &= N \int_{\xi} \mathbb{E}[X_{fj}]\, \mathrm{d}G(\xi) \\ &= Y_j \frac{PD_j}{PD_j + \sum_{d \in \mathcal{D}^F} P\bar{M}_{dj}} + \sum_{d \in \mathcal{D}^F} \bar{Y}_{dj} \frac{PD_j}{PD_j + P\bar{X}_{dj}}, \quad \forall j \in \mathcal{J}, \end{aligned} \tag{15}$$

(iii) Domestic competitiveness can be related to residual profit shifters B_j (through combining equation 15 with an open-economy version of equation 6):

$$B_j = (1 - \varsigma_j) \left(\frac{c_j}{\varsigma_j} \right)^{\frac{\varsigma_j}{\varsigma_j - 1}} \left(\frac{X_j}{PD_j} \right)^{\frac{1}{\sigma_j (1 - \varsigma_j)}}, \tag{16}$$

where c_j stands for the unit price index of a bundle of production inputs assembled using a homothetic Cobb-Douglas technology:

$$c_j \equiv w^{\tilde{\beta}_{jl}} \prod_{m \in J} P_m^{\tilde{\beta}_{jm}},$$

where $\tilde{\beta}_{jl}, \tilde{\beta}_{jm}$ are the share of expenditures (among all expenditures on production inputs) of industry j on input $m \in J$ or labor value-added l, and the domestic consumption price index P_j (in both final and intermediate consumption) is given by:

$$P_j^{1-\sigma_j} \equiv PD_j + \sum_{d \in \mathcal{D}^F} P\bar{M}_{dj}. \tag{17}$$

(iv) The trade balance condition equates manufacturing imports with manufacturing exports plus net exports in the non-manufacturing sector, D (the manufacturing trade deficit):

$$\sum_{j \in \mathcal{J}} Y_j \frac{\sum_{d \in \mathcal{D}^F} P\bar{M}_{dj}}{PD_j + \sum_{d \in \mathcal{D}^F} P\bar{M}_{dj}} = D + \sum_{j \in \mathcal{J}} \sum_{d \in \mathcal{D}^F} \bar{Y}_{dj} \frac{PD_j}{PD_j + P\bar{X}_{dj}}. \tag{18}$$

(v) The residual non-manufacturing sector is produced with constant returns to scale using labor under perfect competition. Domestic value-added in the residual sector is given by

$$wL_{NM} = D + \Pi + T + \beta_{F,NM}(wL),$$

where T is government tariffs (assumed 0 in initial equilibrium), $\beta_{F,NM}$ is the share of final consumption *by private households on the non-manufacturing sector, and Π is net profits in the manufacturing sector given by equation (10).*

(vi) Manufacturing sector payroll is the sum of factor payments in stage II production and stage I capital accumulation:

$$wL_M = \sum_{k\in\mathcal{J}}\left(1 - \sum_{j\in\mathcal{J}}\beta_{kj}\right)X_k - \Pi,$$

where $L_M + L_{NM} = L$ satisfies labor market clearing.

The equilibrium set-up in Definition 1 accommodates different assumptions on how D adjusts to external shocks. Assuming that D is constant allows the use of the trade balance equation (18) to solve wages, w. On the other hand, assuming that D is generated by a perfectly elastic foreign demand for residual services pins down w across counterfactuals; D is simply computed from subtracting new manufacturing imports and exports under fixed w, from the same trade balance equation (18). The results I present in this paper follow this second approach in order to not have wage effects generate spillovers.

C.5 Proof of Proposition 3: Aggregate Impact of Joint Production

Notation: let the home country (the US) be denoted by u, and the set of foreign countries by $\mathcal{D}^F$.

The system of equations in Definition 1 can be log-differentiated to yield a unified system of equations linking endogenous equilibrium variables (price indices, sales, etc) to changes in exogenous variables (changes to domestic scale L, foreign demand $\bar{Y}$, and foreign competitiveness PX, PM).

It is convenient to solve for the equilibrium impact on endogenous variables through their effect on domestic producer price competitiveness, PD_j (an inverse price term defined in Definition 1). The two lines of equation (15) can be interpreted as a supply equals demand market clearing condition. The first line describes a flexible supply-side—the relationship between domestic producer price competitiveness PD_j and market output X_j to sustain firm production incentives under monopolistic competition. The second line describes a downward-sloping demand-curve: the higher is domestic price competitiveness PD_j (the lower are producer prices), the greater is the value of market output. Because both final demand and intermediate demand is unit-elastic (Cobb-Douglas), the only impact of lower prices on market output is through a foreign-market-share-stealing effect.

Demand-side. Log-differentiating the second line of equation (15) yields the following demand-side relationship between sales and changes in gross output X and domestic productivity PD, in matrix algebra:

$$\mathrm{d}\log X = \left(\mathbb{I} - \mathbf{\Omega}^D\right)^{-1}\left(\mathrm{diag}(\lambda^{cpt})\,\mathrm{d}\log PD + \mathrm{d}\log S + \mathrm{diag}(\lambda^X_{uj})\,\mathrm{d}\log PS\right), \tag{19}$$

where λ_j^{cpt} measures the potential for US firms to gain market share from foreign competition in each market:

$$\lambda_j^{cpt} \equiv \sum_{d\in\{u,\mathcal{D}^F\}} \lambda_{dj}^X (1-\lambda_{dj}^M),$$

λ_{dj}^X is the share of the home country's sales going to d, λ_{dj}^M is the share of country d's consumption originating from the home country, the matrix $\mathbf{\Omega}^D$ contains external input-output coefficients denoting the extent to which changes in gross output in other industries k' affect gross output in j:

$$[\mathbf{\Omega}^D]_{jk} \equiv \lambda_{uj}^X (1-\lambda_{j,F}^X) \frac{\beta_{kj} X_k}{\sum_{k'\in\mathcal{J}} \beta_{k'j} X_{k'}},$$

where $\lambda_{j,F}^X$ is the share of final use among all domestic consumption of industry j. I group together the different types of exogenous shocks into two terms: (i) changes to market size faced by US producers, $\mathrm{d}\log S$, and (ii) changes to prices of foreign goods in the US, $\mathrm{d}\log PS$ (while this is also a market size shifter, I single it out here because import prices are a cost shifter in the supply-side equation):

$$\mathrm{d}\log S_j \equiv \lambda_{uj}^X \lambda_{j,F}^X \,\mathrm{d}\log L + \sum_{d\in\mathcal{D}^F} \lambda_{dj}^X \,\mathrm{d}\log \bar{Y}_{dj} - \sum_{d\in\mathcal{D}^F} \lambda_{dj}^X (1-\lambda_{dj}^M)\,\mathrm{d}\log \bar{PX}_{dj},$$

$$\mathrm{d}\log PS_j \equiv -\sum_{d\in\mathcal{D}^F} \lambda_{dj}^{UM} \,\mathrm{d}\log \bar{PM}_j,$$

where λ_{dj}^{UM} is the share of home country's consumption originating from country d,

Note that when the home country is in autarky, $\lambda^{cpt} = 0$, there is no demand-side adjustment of industry output with respect to prices (industry demand is unit-elastic with respect to prices).

Supply-side. I next turn to the supply-side relationship between market size (residual profits) and prices. The first line of equation (15) can be log-differentiated (switching the order of summation across inputs, industries, and firms) to yield

$$\mathbf{\Upsilon}^{-1}\,\mathrm{d}\log X = \mathrm{d}\log B,$$

where $\mathbf{\Upsilon}$ is the aggregate matrix of firm-level cross-price elasticities of supply given by equation (13). I log-differentiate the expression for residual profits B_j in equation (16), open up the production input cost index c_j to reflect intermediate input purchases from manufacturing industries, and replace $\mathrm{d}\log B$ with $\mathrm{d}\log X$ given the above equation to get

$$\mathbf{\Psi}\,\mathrm{d}\log X = -\left(\mathbb{I} - \mathbf{\Omega}^S \;\mathrm{diag}\left(\lambda_{uj}^X\right)\right)\,\mathrm{d}\log PD - \mathbf{\Omega}^S\,\mathrm{d}\log PS, \tag{20}$$

where $\mathbf{\Psi}$ is an inverse matrix of supply elasticities with terms given by

$$[\mathbf{\Psi}]_{jk} \equiv \sigma_j(1-\varsigma_j)[\mathbf{\Upsilon}^{-1}]_{jk} - \mathbf{1}_{j=k} \qquad \forall j,k \in \mathcal{J},$$

and $\mathbf{\Omega}^S$ is matrix of external input-output price-related coefficients given by

$$[\mathbf{\Omega}^S]_{jk} \equiv \beta_{jk}\frac{\sigma_j}{\sigma_k - 1} \qquad \forall j,k \in \mathcal{J}.$$

Equations (20) and (19) represent two systems of equations in two vectors of unknowns (X and PD)—aggregate demand and supply curves. I combine them to express the impact on equilibrium domestic price competitiveness PD changes in terms of arbitrary external shocks collected in $\mathrm{d}\log PS$ and $\mathrm{d}\log S$:

$$\mathrm{d}\log PD = -\left(\mathbb{I} - \mathbf{\Omega}^S \mathrm{diag}(\lambda_u^M) + \mathbf{\Psi}\left(\mathbb{I} - \mathbf{\Omega}^D\right)^{-1} \mathrm{diag}(\lambda^{cpt})\right)^{-1} \times \\ \times \left(\mathbf{\Psi}\left(\mathbb{I} - \mathbf{\Omega}^D\right)^{-1} \mathrm{d}\log S + \left(\mathbf{\Omega}^S + \mathbf{\Psi}\left(\mathbb{I} - \mathbf{\Omega}^D\right)^{-1} \mathrm{diag}(\lambda_u^X)\right) \mathrm{d}\log PS\right). \quad (21)$$

Proposition 3 follows the fact that the PPI is defined as $(1-\sigma_j)\,\mathrm{d}\log\mathcal{P}_j = \mathrm{d}\log PD_j$ and from setting $\mathrm{d}\log PS = 0$ in the above expression (so that the only exogenous shock is to market size).

C.6 Calibration to US Manufacturing Sector

Notation: $\mathcal{D}^F = \{c, r\}$ refer to China and the rest-of-the-world composite respectively.

Data in the Initial Equilibrium. These equilibrium definitions allow me to impute consumption expenditure shares β_j, the manufacturing deficit D, all price competitiveness indices, and foreign expenditures $\bar{Y}_{c,j}, \bar{Y}_{r,j}$ given US and world trade and industry level data in 2017. I introduce the data I use from public data in 2017:

1. Data on gross output by manufacturing industry, X_j come from the BEA in 2017.
2. I hold the number of total manufacturing firms, N, fixed, at 318,000.
3. Data on β_{jk} and ς_k come from the 2012 BEA I/O tables (the 2017 tables are not yet available).[57]
4. Trade data in 2017 on US imports and exports by country and industry (after mapping HS10 to BEAX) come from the US Census Bureau (made available by Schott (2008)).[58]
5. World trade data in 2017 by industry and country come from BACI Comtrade.

[57] There are a few industries where implied input-output use shares are so large that final use is predicted to be negative. I adjust input-output shares downward by a proportional factor for that industry until final use is at least 2% of gross consumption.

[58] There are a few industries where US exports is higher than measures of gross output in BEA data. I harmonize the two data sources by adjusting gross output, X_j, to be at least 1% higher than gross exports.

Variables in the Model. Using the trade data, I compute λ^X_{dj} as the share of US firms' total sales in industry j going to destination $d \in \{u, r, c\}$, λ^M_{dj} is the share of destination $d \in \{u, r, c\}$'s consumption of industry j on goods sold by the US. These trade shares are important; note that all the ratios of price competitiveness in the definition of equilibrium can be written as these observable trade shares.

I compute industry gross expenditures as $Y_j = \frac{X_j \lambda^X_{uj}}{\lambda^M_{uj}}$.

Using the estimated micro parameters, I repeat the same macro inversion steps as in the structural estimation to estimate macro variables α, B, Z in 2017. I use 1997 expenditure shares on knowledge inputs categories $m \in \mathcal{M}^{KLG}$ by each industry j combined with 2017 output data to impute expenses on knowledge inputs M_{jm} used in the inversion. With these macro variables on hand I compute net profits in the manufacturing sector Π using equation (10).

I compute the manufacturing deficit as the difference between total consumption and total output: $D = \sum_{j\in\mathcal{J}} Y_j - \sum_{j\in\mathcal{J}} X_j$. I normalize the wage w to 1 by choosing an appropriate unit in which to measure efficiency-adjusted labor, whereby

$$L = GDP - \Pi,$$

where GDP is 19.4 trillion in 2017. The share of consumption on residual services is then given by:

$$1 - \beta_S = \frac{\sum_{j\in\mathcal{J}} \left(Y_j - \sum_k \beta_{kj} X_k\right)}{L}.$$

I compute final consumption shares in manufacturing, $\beta_{F,k}$, as:

$$\beta_{F,k} = \frac{Y_k - \sum_j \beta_{kj} X_k}{L}.$$

Foreign demand in the model is given by $\bar{Y}_{r,j} \lambda^M_{rj} = EX_{urj}$ where EX_{urj} is US exports to destination r in industry j. An identical expression pins down $\bar{Y}_{c,j}$.

C.7 Counterfactuals: Model Responses to Small Shocks

C.7.1 Quantifying the Impact of Joint Production

I use the calibrated model and Proposition 3 to quantify the impact of small shocks on equilibrium producer price indices. The main results described in Figure 6 are computed as follows. I express the impact as:

$$\mathrm{d}\log PD = \boldsymbol{\Xi}\, \mathrm{d}\log S,$$

where Ξ is a matrix defined below, taking on different values across the scenarios described in the main body of text.

$$\Xi = -\left(\mathbb{I} - \mathbf{\Omega}^S \text{diag}(\lambda_u^M) + \mathbf{\Psi}\left(\mathbb{I} - \mathbf{\Omega}^D\right)^{-1} \text{diag}(\lambda^{cpt})\right)^{-1} \mathbf{\Psi}\left(\mathbb{I} - \mathbf{\Omega}^D\right)^{-1}$$

For each scenario (associated with a Ξ), I decompose Ξ into diagonal and off-diagonal elements, respectively, as

$$\Xi = \Xi^{DIAG} + \Xi^{CROSS}.$$

The values of $\lambda_j^X \equiv X_j/X$ are shares of industry j output among total manufacturing output X. I use these shares to collapse the vector of price competitiveness effects $\text{d}\log PD_j$ into an aggregate producer price index. I compute, for example, the direct effect of a proportional change in foreign market size across all industries $\mathcal{J}$ on the PPI as:

$$\text{d}\log PPI^{DIRECT} = \left(\frac{\lambda^X}{1-\sigma}\right)' \times \Xi^{DIAG} \times (\lambda_r^X + \lambda_c^X), \tag{22}$$

and equivalently for component due to cross-industry spillovers using the Ξ^{CROSS} matrix above instead of Ξ^{DIAG}.

Table 13 describes the numbers behind producing Figure 6. Because prices fall in response to scale, I invert the PPI to convert negative numbers into positive numbers, so the responses can be construed of as responses of model-consistent productivity (inverse prices) to exogenous shocks. I compute the effects of a 1% proportional rise in foreign demand (across all industries) on the manufacturing PPI and gross output. To compute the effect on industry gross output, I solve out for the demand and supply equations (20) and (19) to express $\text{d}\log X$ in terms of exogenous shocks instead of $\text{d}\log PD$. The change in manufacturing sector gross output is computed using the same λ_j^X-weighted average over industry-level changes. I further decompose the effect of spillovers (shown in Figure 6) into positive and negative contributions, to emphasize that the majority of gross cross-industry elasticities are price-decreasing (positive impact on inverse-PPI).

I compute the share of the total price impact due to the joint production matrix (non-zero cross-price elasticities of supply) by differencing the PPI impact between scenarios (b) and (a), and likewise for scenarios (d) and (c) and expressing this difference as a share of the total PPI effect (in (b) and (d), respectively).

Table 14 presents results for the main counterfactual, scenario (b), when foreign demand shocks occur industry-by-industry rather than manufacturing sector-wide. It shows the top and bottom industries that transmit industry shocks into other parts of the manufacturing sector. I compute the proportion of PPI response that occurs as a result of spillovers relative to the total effect (spillovers + direct) whenever the industry receives an industry-specific demand shock.[59] A positive spillover

[59]The top industries listed do not indicate the industries where demand shocks generate the highest total aggregate productivity gains, nor industries where the total elasticity of productivity to gross output is

Table 13: Effect of a 1% Increase in Foreign Demand on Manufacturing PPI and Output

Scenario	(a)	(b)	(c)	(d)
Response of Inverse PPI (%)	0.066	0.082	0.636	1.410
Direct	0.066	0.066	0.067	0.134
Positive Spillovers	0	0.018	0.526	1.275
Negative Spillovers	0	-0.002	0	0.000
Response of Gross Output (%)	0.387	0.416	1.976	3.922
Direct	0.387	0.392	0.515	0.581
Positive Spillovers	0	0.032	1.460	3.342
Negative Spillovers	0	-0.003	0	0.000
Elasticity of Inverse PPI with respect to Gross Output	0.17	0.20	0.32	0.36
share due to internal cross-price elasticities of supply	/	0.20	/	0.55

Notes: This table depicts the effect on manufacturing inverse PPI and output in the US due to a proportional 1% foreign demand shock (across all industries). The sub-rows decompose the total effects into those accruing due to direct, own-industry responses versus cross-industry spillovers. A 'positive' spillover refers to a decrease in the PPI. The four columns depict four scenarios corresponding to different versions of the underlying economy: (a) only internal scale economies (along the own-industry), (b) both internal scale and scope economies due to the full joint production matrix, (c) only internal scale economies combined with external I/O linkages, (d) both internal scale and scope economies combined with external I/O linkages. The elasticities in Figure 6 correspond to the direct and spillover inverse PPI effects scaled by the total effect on gross output. The bars for each scenario sum to the penultimate row, the total elasticity of the PPI with respect to gross output. The last row operates across scenarios: comparing the contribution to the PPI response when internal cross-price elasticities of supply are allowed to be non-zero. I take the difference in PPI between (b) and (a) divided by the PPI response in (b). I do the analogous computation for scenario (d) by comparing to (c).

Table 14: Top and Bottom Industries by Positive Spillovers Generated

BEAX	Description	Share
334514	Totalizing fluid meter and counting device	0.71
3122A0	Tobacco product manufacturing	0.71
334510	Electromedical and electrotherapeutic apparatus manufacturing	0.60
334516	Analytical laboratory instrument	0.58
33451B	Watch, clock, and other measuring and controlling device manufacturing	0.57
33641A	Propulsion units and parts for space vehicles and guided missiles	0.52
33461X	Manufacturing and reproducing magnetic and optical media	0.50
339115	Ophthalmic goods manufacturing	0.46
334511	Search, detection, and navigation instruments	0.45
325620	Toilet preparation manufacturing	0.44
31161A	Animal (except poultry) slaughtering, rendering, and processing	0.06
326160	Plastics bottle manufacturing	0.06
314110	Carpet and rug mills	0.06
311920	Coffee and tea manufacturing	0.06
312140	Distilleries	0.05
337215	Showcase, partition, shelving, and locker manufacturing	0.05
311221	Wet corn milling	0.03
33142X	Copper rolling, drawing, extruding and alloying	0.02
327992	Ground or treated mineral and earth manufacturing	0.02
311930	Flavoring syrup and concentrate manufacturing	-0.06

Notes: This table depicts the top and bottom 10 industries in terms of the proportion of aggregate PPI impact (due to a marginal demand shock in that industry) that accrues due to net cross-industry spillovers. A positive spillover means that a positive demand shock in that industry leads to price *declines* in other manufacturing industries. These effects are computed using equation (13); the numbers here correspond to the scatterplot in Figure 7. Results are generated as an average of 100 monte carlo simulations of the model's draws of firm productivities to wash out sampling variation.

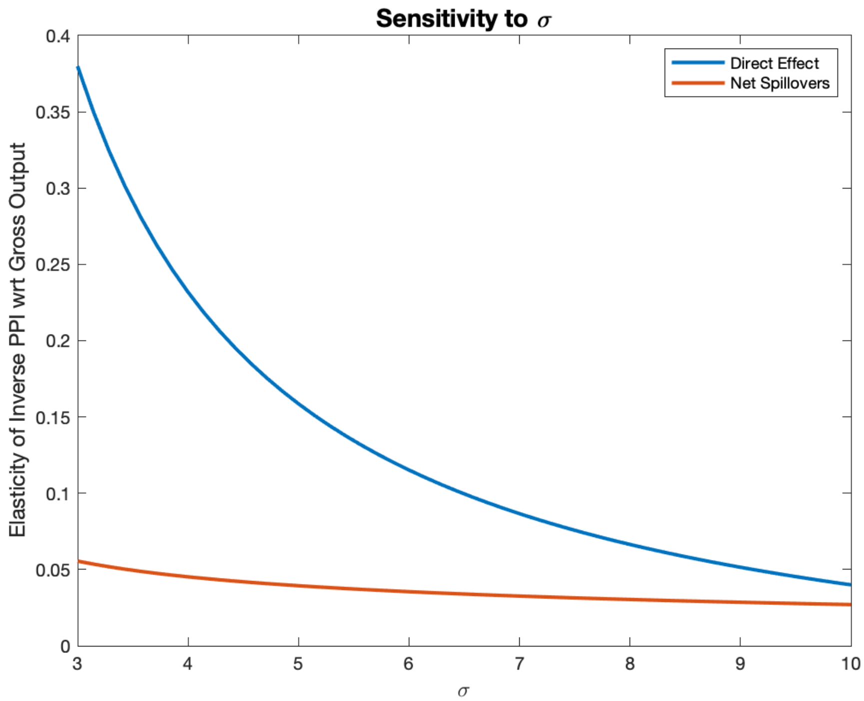
means that a positive demand shock in that industry leads to price *declines* in other manufacturing industries.

C.7.2 Robustness to Alternative values of σ

I explore the sensitivity of the results in Table 13 to alternative values of σ_j, against the benchmark of $\sigma_j = 5\ \forall j$. I consider three different exercises. First, I vary the absolute value of σ but continue to hold it constant across industries. Figure 8 shows how the elasticity of the inverse PPI with respect to gross output varies with values of $\sigma \in (3, 10)$, decomposing the total PPI response to the direct, own-industry effect versus net spillovers. While the direct effect decreases as expected with the value of σ_j, the indirect effect does so at a much slower pace, contributing between 3 and

highest. The top 5 industries by total aggregate productivity gains are aircraft manufacturing, petroleum refineries, other motor vehicle parts, light truck and utility vehicles, and broadcast and wireless communications equipment.

Figure 8: Elasticity of Inverse PPI to Gross Output under alternative values of σ



Notes: This graph explores the sensitivity of the results in Figure 6 (where $\sigma_j = 5\ \forall j$) to alternative common values of σ across industries. I decompose the elasticity of inverse PPI to output in scenario (b) (no I/O linkages) into the direct effect in the blue line and an indirect (cross-industry) effect in the orange line.

5 percentage points to the elasticity aggregate productivity to gross output across the entire range of values of σ.

In the next two calibrations, I allow σ_j to vary across industries j by: (i) assuming that profit shares (gross operating profits) in each industry are equal to $\frac{1}{\sigma_j}$ (as would be true in a case with constant returns to scale, monopolistically competitive firms, and sunk entry costs paid in some pre-period), and (ii) targeting the same own-sector returns to scale as in Bartelme et al. (2019). While assumption (i) is ad-hoc (and not consistent with the model), the values of σ_j serve as a useful benchmark as they appear in other papers. Assumption (ii) is, however, model-consistent. In this scenario, I first generate a mapping from BEAX to the two-digit manufacturing sectors in Table 1 of Bartelme et al. (2019), denoted by s. I allow for as many differences across σ_j as there are sectors s, so all σ_j is the same within a sector but different across sectors. I then solve for the values of σ_s that would generate the following relationship between sectoral price indices and sectoral size (from combining equations 22 and 20).

$$\frac{\mathrm{d}\log PPI_s}{\mathrm{d}\log X_s} = -\gamma_s^{BCDR} = -\sum_{j\in s}\frac{\lambda_j^X}{1-\sigma_s}\sum_{k\in s}\Psi_{jk}, \qquad \forall s,$$

Table 15: Effect of a 1% Increase in Foreign Demand on Manufacturing PPI and Output: Robustness under different values of σ_j

Calibrated values of σ_j	Baseline (=5)		(i) Profit Share		(ii) BCDR	
Scenario	(b)	(d)	(b)	(d)	(b)	(d)
Response of Inverse PPI (%)	0.082	1.410	0.085	0.417	0.064	1.073
Direct	0.066	0.134	0.069	0.094	0.050	0.101
Positive Spillovers	0.018	1.275	0.018	0.323	0.016	0.972
Negative Spillovers	-0.002	0.000	-0.002	0.000	-0.002	0.000
Response of Gross Output (%)	0.416	3.922	0.408	1.639	0.395	3.299
Direct	0.387	0.581	0.372	0.454	0.365	0.524
Positive Spillovers	0.032	3.342	0.039	1.185	0.032	2.775
Negative Spillovers	-0.003	0.000	-0.004	0.000	-0.003	0.000
Elasticity of Inverse PPI w.r.t. Gross Output	0.20	0.36	0.21	0.25	0.16	0.33
share due to internal cross-price elasticities of supply	0.20	0.55	0.19	0.43	0.23	0.52

Notes: This table explores the sensitivity of results in Table 13 and Figure 6 to alternative calibrations of σ_j that vary across industries. Over the rows of each column I compute the effect on inverse PPI and output in the US due to a proportional 1% foreign demand shock (across all industries). The sub-rows decompose the total effects into those accruing due to direct, own-industry responses versus spillovers across industries. A 'positive' spillover refers to a decrease in the PPI. The penultimate row divides the total response of inverse PPI by the total response of output. The last row operates across scenarios: comparing the contribution to the PPI response when internal cross-price elasticities of supply are allowed to be non-zero. I take the difference in PPI between (b) and (a) divided by the PPI response in (b). I do the analogous computation for scenario (d) by comparing to (c). Across the columns, I alter the values of σ and the scenario. The first two columns reproduce key results in Table 13: results obtained under the baseline assumption of $\sigma_j = 5\ \forall j$, for scenarios (b) (no I/O linkages) and (d) (incl. I/O linkages). The next set of two columns repeat the baseline calculations but using σ_j calibrated to match the profit share in each industry assuming constant returns to scale, and the final set of two columns repeat the baseline calculations but using σ_s calibrated (at the 2-digit manufacturing level) to match sectoral level returns to scale in Bartelme et al. (2019).

where γ_s^{BCDR} are estimates of scale elasticities in Table 1 of Bartelme et al. (2019).[60]

Table 15 displays how the baseline results in Table 13 change with respect to the two different calibrations, both with and without I/O linkages. The different calibrations with heterogeneous σ_j do not alter the main quantitative message that spillovers due to joint production are large. In the benchmark model set-up without input-output links (b), internal cross-price elasticities arising from joint production accounts for between 19% (scenario (i), Profit Share) and 23% (scenario (ii), BCDR) of the total elasticity of the inverse PPI with respect to gross output. Whereas direct, own-industry effects under scenario (b) of the BCDR calibration generate an aggregate scale elasticity of 0.13 (=0.050/0.395), accounting for spillovers induced by internal cross-price elasticities of supply brings this up to 0.16 (=0.064/0.395).

[60]Note that this procedure does attribute (correctly) the cross-industry spillovers across industries within a sector to sectoral economies of scale. The only productivity response that would be missed in BCDR are cross-sectoral spillovers.

C.8 Counterfactuals: Model Responses to Arbitrary Large Shocks

C.8.1 Tariff Shocks

The model accommodates different types of counterfactual shocks. I consider new tariffs imposed by the US on imports from China, denoted by τ_{cuj}, and import tariffs imposed by China on imports from the US, denoted by τ_{ucj}. I model tariffs $\tau \geq 1$ as ad-valorem, so that

1. The change in Chinese price competitiveness in the US is $\hat{\hat{PM}}_{cj} = \tau_{cuj}^{1-\sigma_j}$.
2. The change in US price competitiveness in China can be modeled as $\hat{\hat{PX}}_{cj} = \tau_{ucj}^{\sigma_j-1}$. Tariffs also cause take-home revenues of firms to fall to $\frac{1}{\tau_{ucj}}$ of tax-inclusive sales. This can be reflected by a change in $\hat{\hat{Y}}_{cj} = \tau_{ucj}^{-1}$.
3. US import tariff revenues are given by

$$T' = \sum_j \frac{\tau_{cuj} - 1}{\tau_{cuj}} \tau_{cuj}^{1-\sigma_j} \lambda_{cj}^{UM} \hat{P}_j^{\sigma_j-1},$$

   I assume that pre-existing tariffs on Chinese imports are zero. If they are non-zero, the new tariffs change infra-marginal tariff revenues and the calculation needs to be revised. I assume that Chinese tariffs on US goods are taken out of the system and do not go towards increasing market demand $\bar{Y}_{c,j}$.

C.8.2 Solving for the Model's Responses in Exact Changes

For any set of counterfactual exogenous shocks, the system of equations admits a new solution for PD_j and w. I solve the system of equations in terms of exact hat changes. Specifically, for any guess of $\hat{PD}_j$ and $\hat{w}$, I can compute

$$\hat{B}_j = \hat{c}_j^{\frac{\varsigma_j}{\varsigma_j-1}} \left(\frac{\hat{X}_j}{\hat{PD}_j} \right)^{\frac{1}{\sigma_j(1-\varsigma_j)}},$$

where $\hat{c}_j$ is given by

$$\hat{c}_j = \hat{w}^{\tilde{\beta}_{lj}} \prod_{m \in J} \hat{P}_m^{\tilde{\beta}_{jm}},$$

and $\hat{P}_j$ is the change in the domestic price index given by

$$\hat{P}_j^{1-\sigma_j} = \hat{PD}_j \lambda_{uj}^{UM} + \hat{\hat{PM}}_{cj} \lambda_{cj}^{UM} + \hat{\hat{PM}}_{rj} \lambda_{rj}^{UM},$$

and $\hat{X}_j$ is given by

$$\hat{X}_j X_j = Y_j' \hat{PD}_j \lambda_{u,j}^{M} \hat{P}_j^{\sigma_j-1} + \bar{Y}_{c,j} \hat{\hat{Y}}_{c,j} \hat{PD}_j \lambda_{cj}^{M} \hat{P}_{chn,j}^{\sigma_j-1} + \bar{Y}_{r,j} \hat{\hat{Y}}_{r,j} \hat{PD}_j \lambda_{rj}^{M} \hat{P}_{row,j}^{\sigma_j-1},$$

and $\hat{P}_{row,j}$ is the change in the rest-of-world consumption price index given by

$$\hat{P}_{row,j}^{1-\sigma_j} = P\hat{D}_j \lambda_{rj}^M + P\hat{\hat{X}}_{rj}(1-\lambda_{rj}^M),$$

$\hat{P}_{chn,j}$ is the change in the consumption price index in China given by

$$\hat{P}_{chn,j}^{1-\sigma_j} = P\hat{D}_j \lambda_{cj}^M + P\hat{\hat{X}}_{cj}(1-\lambda_{cj}^M),$$

and finally the new vector of gross expenditures Y_j' can be inverted from

$$Y_j' = \sum_k \beta_{kj}\left(\hat{X}_k X_k\right) + \beta_{F,j}\hat{w}L\hat{L},$$

where T' is tariff revenues defined above.

To evaluate the guess I use a system of J equations equal to deviations between industry sales as computed above, X_j', and the implied industry sales (by solving the firm's problem) given by equation 15 under the new B_j'. I also use the trade balance condition:

$$\sum_j Y_j' = D' + \sum_j \hat{X}_j X_j,$$

to either pin down D' when $\hat{w} = 1$ (foreign demand for residual services is assumed to be perfectly elastic), or to solve for $\hat{w}$ when D is held exogenous (as is more typical in trade counterfactuals). I find that a gradient based optimization algorithm works very well with this system of equations.

Equilibrium Changes. Throughout counterfactuals presented in Table 6, I compute several changes in macroeconomic variables of interest:

1. The change in the manufacturing CPI (consumer price deflator) is

$$\prod_j \hat{P}_j^{\beta_{F,j}}$$

2. The change in the manufacturing CPI excluding the domestic response of productivity is

$$\prod_j \left(\hat{P}_j^{CTF}\right)^{\beta_{F,j}},$$

where

$$\left(\hat{P}_j^{CTF}\right)^{1-\sigma_j} = \lambda_{uj}^{UM} + P\hat{\hat{M}}_{cj}\lambda_{cj}^{UM} + P\hat{\hat{M}}_{rj}\lambda_{rj}^{UM}.$$

3. Expressions for the change in imports, US output and US exports in each industry, tariff revenues and the deficit can also be computed directly given the equations above.

C.8.3 Alternative Tariffs

I compute the vector of alternative tariffs as follows. First, local effects of external shocks on the CPI can be computed from log-differentiating the price index equation (17):

$$\mathrm{d}\log CPI = \sum_j \beta_j \,\mathrm{d}\log P_j = \sum_j \frac{\beta_j}{1-\sigma_j}\left(\lambda^M_{uj}\,\mathrm{d}\log PD_j - \mathrm{d}\log PS_j\right),$$

where the change in foreign prices in the US is given by $\mathrm{d}\log PS_j = \lambda^{UM}_{cj}(\sigma_j - 1)\,\mathrm{d}\log\tau_j$. I use equation (21) to replace $\mathrm{d}\log PD_j$ above with its value in terms of the exogenous tariff shock, $\{\mathrm{d}\log\tau_j\}_j$.

Second, the change in imports from China is given by

$$\mathrm{d}\log IM_c = \sum_j \lambda_{cj}\left(-\lambda^{UM}_{uj}\,\mathrm{d}\log PD_j - (1-\lambda^{UM}_{cj})(\sigma_j - 1)\,\mathrm{d}\log\tau_j + \mathrm{d}\log Y_j\right),$$

where λ_{cj} is the share among all US imports from China that come form industry j, and again $\mathrm{d}\log Y_j$ and $\mathrm{d}\log PD_j$ can be written in terms of $\mathrm{d}\log\tau_j$ using the equilibrium relationships surrounding equation (21).

A marginal tariff in each industry thus generates a cost (higher CPI) versus a benefit (reducing imports from China, assuming that it is a policy-making objective). I manipulate the matrix of relationships above to yield

$$\mathring{\tau}_k = \frac{\frac{-\,\mathrm{d}\log IM_c}{\mathrm{d}\log\tau_k}}{\frac{\mathrm{d}\log CPI}{\mathrm{d}\log\tau_k}}, \quad \forall k \in \mathcal{J},$$

which expresses the effect of a marginal tariff in industry k towards achieving the policy target (reducing imports from China) per unit of increase in the CPI. Industries k with a higher $\mathring{\tau}_k$ should have higher tariffs (due to their higher benefit / cost ratio), but exactly how much higher is not exact given the non-linear structure of the model. I use this starting vector of alternative tariffs, $\mathring{\tau}$, as the basis for a search over various scaling parameters $\beta \geq 0$:

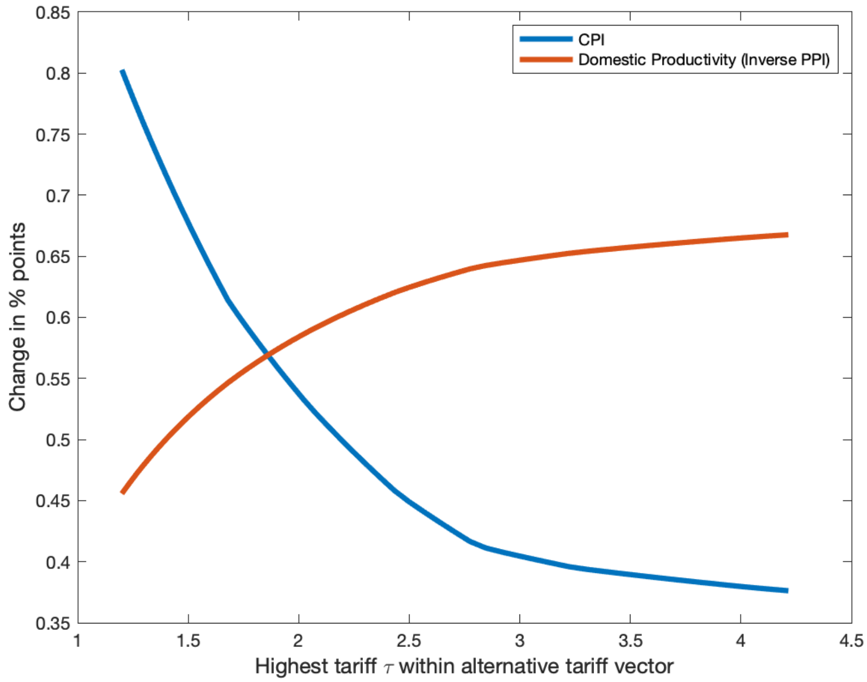
$$\tau_j = \max(1, \alpha\mathring{\tau}_j^{\beta}),$$

where the α is chosen so that the alternative tariffs τ_j reduce Chinese imports by 41%, the same amount as uniform tariffs of 20% would. Intuitively, values of β adjust the skew / bias that is applied to the tariffs identified by the local propagation matrix as generating higher benefit / cost ratios.[61]

Figure 9 displays the impact of alternative tariffs on the CPI and the PPI, across varying degrees of industry bias in tariffs on the x-axis (induced by values of β). For each value of β, the parameter

[61] While the values $\mathring{\tau}$ are identified using a local propagation matrix, the computed effects shown in Figure 9 and Table 6 are fully non-linear and not local approximations.

Figure 9: Impact of Alternative Tariff Policies on US Manufacturing CPI and Productivity



Notes: This graph displays the impact on the manufacturing CPI (in blue) and PPI (in orange) from a range of tariff policies that are designed to achieve the same reduction in US imports from China (of 41%). Tariff policies range across the x-axis in terms of their degree of bias towards the vector $\mathring{\tau}$ identified by the model's propagation matrix. The bias is summarized as the highest value of τ among the vector. A highest value of 1.2 (the initial point) reflects the benchmark counterfactual of blanket 20% tariffs.

α is chosen to scale the set of tariffs so that the overall impact on reducing Chinese imports is 41%. The left-most point, where tariffs of $\tau = 1.2$ are symmetric across industries, corresponds to Column (2) of Table 6. Intuitively, as bias increases, more concentrated tariffs are required in order to achieve the same reduction in Chinese imports. Throughout these exercises I restrict tariffs to be weakly positive (so import subsidies are ruled out).

C.8.4 China Import Tariff Counterfactual: Model with Input-Output Linkages

Finally, in Table 16, I repeat the China import tariff counterfactuals under the extension of the model with input-output linkages. First, in the neoclassical setting under constant returns to scale (Column 1), I find that input-output linkages generate higher increases in the CPI relative to the case without I/O linkages. Not only are prices of directly imported goods higher, but domestic goods prices also increase due to higher input costs. Column (2), which displays results from my model, highlights additional competing forces coming from joint production. While import taxes raise the cost of foreign imported inputs, substitution towards the use of domestic intermediate goods generates a scale effect which lowers the cost of domestic inputs. These domestic price effects are amplified by further success in export markets (the foreign demand for US exports is elastic), so that the net effect on upstream input prices faced by US producers in any industry is theoretically ambiguous. I find, in this calibration, that the domestic scale effect dominates. The results in Column (2) under input-output linkages actually reduce welfare losses of tariffs compared to the main results presented in column (2) of Table 6 (without input-output linkages). The improved success of domestic producers is reflected in an increase in net profits (equal to 0.28% of gross manufacturing output) and a much larger reduction in the trade deficit. Finally, column (3) displays results from applying a similar process of searching for alternative import tariffs that mitigate the reduction in the CPI while achieving the same reduction in imports from China. The results demonstrate that there are pure gains from import protection in this environment with scale and scope economies, exogenous foreign prices, and wages pinned down by a residual (non-manufacturing) sector. These results are sensitive, however, to assumptions on the foreign import demand elasticity (implicit behind this setup is the ability of US producers to steal market share from foreign competitors in foreign markets). Inelastic foreign demand would curtail the amplification of domestic scale effects and would turn the impact on the CPI back to positive territory.

Table 16: Model with external I/O links: Effects on the US Economy from US Tariffs on Chinese Imports

Model (with input-output linkages) Import tariffs on Chinese imports	(1) Neoclassical CRS 20% Uniform	(2) Scale + Scope 20% Uniform	(3) Scale + Scope Alternative
Change (%)			
Consumer Price Index (CPI)	1.69	0.11	-2.68
Producer Price Index (PPI)	0.58	-2.12	-6.10
Imports from China	-38.7	-40.8	-40.8
Imports from Rest of World	8.9	4.6	-4.2
US Output	0.6	7.4	20.2
US Exports	2.6	7.5	26.0
Manufacturing Trade Deficit	-3.0	-32.5	-86.3
As Share of Manufacturing Output (%)			
New Manufacturing Profits	0	0.28	0.72
New Tariff Revenues	0.89	0.86	0.15

Notes: This table extends the results in Table 6 to an environment with external input-output linkages. It presents estimates of the impact of two different sets of tariffs on outcomes in the US economy, under two different model settings calibrated to match US national industry-level aggregates (industry output and trade patterns) in 2017. Column (1) presents results from a 20% increase in tariffs on Chinese imports across all industries, under a neoclassical model with constant returns to scale and perfect competition. Column (2) compares the impact of the same set of tariffs to my model with economies of scale and scope. Column (3) presents results from alternative tariffs predicted by my model that seek to minimize the CPI impact while achieving the same reduction in total imports from China. See Definition 1 for a characterization of the open economy equilibrium and Appendix Section C.8.3 for details on how I compute alternative tariffs.